

# 1 Short help on Parks-McClellan design of FIR Low Pass Filters using Matlab

The design of an FIR filter using Parks-McClellan algorithm is a two-step process. First, you need to use the *firpmord* command to estimate the order of the optimal Parks-McClellan FIR filter to meet your design specifications. The syntax of the command is as follows:

$$[n,fo,mo,w]=firpmord(f,m,dev)$$

$f$  is the vector of band frequencies. For a low pass filter,  $f=[wp \ ws]$  where  $wp$  is the upper edge of the passband and  $ws$  is the lower edge of the stopband. The vector  $m$  contains the desired magnitude response values at the passbands and the stopbands of the filter. Since a lowpass filter consists of a passband followed by a stopband,  $m$  has two entries. Namely,  $m=[1 \ 0]$  because you would like the magnitude response to be equal to 1 in the passband and equal to 0 in the stopband. The vector  $dev$  has the maximum allowable deviations of the magnitude response of the filter from the desired magnitude response. It has the same number of entries as there are  $m$ . Thus, for a low-pass filter it has two entries and is equal to  $[passband \ ripple, \ stopband \ ripple]$ . After you specify the vectors  $f$ ,  $m$  and  $dev$ , you can run the *firpmord* command in the syntax given above to compute the order of the filter  $n$ . The *firpmord* command also outputs the resulting  $fo$  and  $mo$  vectors. In actually designing your filter using *firpm*, you should use these two vectors instead of  $f$  and  $m$ . The second stage is the actual design of the filter, using the *firpm* command. After running *firpmord* and finding the  $n$ ,  $fo$  and  $mo$ , type

$$b=firpm(n,fo,mo)$$

to find the impulse response  $b$  of the Parks-McClellan FIR filter you need to design. The vector  $b$  contains the coefficients for the  $Z$ -transform of your filter  $H(z)$ . That is,

$$H(z) = b(1) + b(2)z^{-1} + b(3)z^{-2} + \dots + b(n+1)z^{-n}$$

This concludes the design of your filter. The *firpmord* and *firpm* can be used to design also highpass, bandpass and multiband filters. See the Matlab help for details.

## 2 Matlab Help on firpmord

**FIRPMORD** Parks-McClellan optimal equiripple FIR order estimator.

$[N,Fo,Ao,W] = \text{FIRPMORD}(F,A,DEV,Fs)$  finds the approximate order  $N$ , normalized frequency band edges  $Fo$ , frequency band magnitudes  $Ao$  and weights  $W$  to be used by the FIRPM function as follows:

$$B = \text{FIRPM}(N,Fo,Ao,W)$$

The resulting filter will approximately meet the specifications given

by the input parameters  $F$ ,  $A$ , and  $DEV$ .  $F$  is a vector of cutoff frequencies in Hz, in ascending order between 0 and half the sampling frequency  $F_s$ . If you do not specify  $F_s$ , it defaults to 2.  $A$  is a vector specifying the desired function's amplitude on the bands defined by  $F$ . The length of  $F$  is twice the length of  $A$ , minus 2 (it must therefore be even). The first frequency band always starts at zero, and the last always ends at  $F_s/2$ . It is not necessary to add these elements to the  $F$  vector.  $DEV$  is a vector of maximum deviations or ripples (in linear units) allowable for each band.  $DEV$  must have the same length as  $A$ .

$C = \text{FIRPMORD}(F,A,DEV,FS,'cell')$  is a cell-array whose elements are the parameters to  $\text{FIRPM}$ .

#### EXAMPLE

Design a lowpass filter with a passband-edge frequency of 1500Hz, a stopband-edge of 2000Hz, passband ripple of 0.01, stopband ripple of 0.1, and a sampling frequency of 8000Hz:

```
[n,fo,mo,w] = firpmord( [1500 2000], [1 0], [0.01 0.1], 8000 );
b = firpm(n,fo,mo,w);
```

This is equivalent to

```
c = firpmord( [1500 2000], [1 0], [0.01 0.1], 8000, 'cell');
b = firpm(c{:});
```

CAUTION 1: The order  $N$  is often underestimated. If the filter does not meet the original specifications, a higher order such as  $N+1$  or  $N+2$  will.  
CAUTION 2: Results are inaccurate if cutoff frequencies are near zero frequency or the Nyquist frequency.

See also  $\text{FIRPM}$ ,  $\text{KAISERORD}$ .

## 3 Matlab Help on `firpm`

$\text{FIRPM}$  Parks-McClellan optimal equiripple FIR filter design.

$B = \text{FIRPM}(N,F,A)$  returns a length  $N+1$  linear phase (real, symmetric coefficients) FIR filter which has the best approximation to the desired frequency response described by  $F$  and  $A$  in the minimax sense.  $F$  is a vector of frequency band edges in pairs, in ascending order between 0 and 1. 1 corresponds to the Nyquist frequency or half the sampling frequency. At least one frequency band must have a non-zero width.  $A$  is a real vector the same size as  $F$  which specifies the desired amplitude of the frequency response of the resultant filter  $B$ .

The desired response is the line connecting the points  $(F(k), A(k))$  and  $(F(k+1), A(k+1))$  for odd  $k$ ; FIRPM treats the bands between  $F(k+1)$  and  $F(k+2)$  for odd  $k$  as "transition bands" or "don't care" regions. Thus the desired amplitude is piecewise linear with transition bands. The maximum error is minimized.

For filters with a gain other than zero at  $F_s/2$ , e.g., highpass and bandstop filters,  $N$  must be even. Otherwise,  $N$  will be incremented by one. Alternatively, you can use a trailing 'h' flag to design a type 4 linear phase filter and avoid incrementing  $N$ .

$B = \text{FIRPM}(N, F, A, W)$  uses the weights in  $W$  to weight the error.  $W$  has one entry per band (so it is half the length of  $F$  and  $A$ ) which tells FIRPM how much emphasis to put on minimizing the error in each band relative to the other bands.

$B = \text{FIRPM}(N, F, A, \text{'Hilbert'})$  and  $B = \text{FIRPM}(N, F, A, W, \text{'Hilbert'})$  design filters that have odd symmetry, that is,  $B(k) = -B(N+2-k)$  for  $k = 1, \dots, N+1$ . A special case is a Hilbert transformer which has an approx. amplitude of 1 across the entire band, e.g.  $B = \text{FIRPM}(30, [.1 .9], [1 1], \text{'Hilbert'})$ .

$B = \text{FIRPM}(N, F, A, \text{'differentiator'})$  and  $B = \text{FIRPM}(N, F, A, W, \text{'differentiator'})$  also design filters with odd symmetry, but with a special weighting scheme for non-zero amplitude bands. The weight is assumed to be equal to the inverse of frequency times the weight  $W$ . Thus the filter has a much better fit at low frequency than at high frequency. This designs FIR differentiators.

$B = \text{FIRPM}(\dots, \{\text{LGRID}\})$ , where  $\{\text{LGRID}\}$  is a one-by-one cell array containing an integer, controls the density of the frequency grid. The frequency grid size is roughly  $\text{LGRID} * N / 2 * \text{BW}$ , where  $\text{BW}$  is the fraction of the total band interval  $[0, 1]$  covered by  $F$ .  $\text{LGRID}$  should be no less than its default of 16. Increasing  $\text{LGRID}$  often results in filters which are more exactly equiripple, at the expense of taking longer to compute.

$[B, \text{ERR}] = \text{FIRPM}(\dots)$  returns the maximum ripple height  $\text{ERR}$ .

$[B, \text{ERR}, \text{RES}] = \text{FIRPM}(\dots)$  returns a structure  $\text{RES}$  of optional results computed by FIRPM, and contains the following fields:

$\text{RES.fgrid}$ : vector containing the frequency grid used in  
the filter design optimization  
 $\text{RES.des}$ : desired response on  $\text{fgrid}$

RES.wt: weights on fgrid  
 RES.H: actual frequency response on the grid  
 RES.error: error at each point on the frequency grid (desired - actual)  
 RES.iextr: vector of indices into fgrid of extremal frequencies  
 RES.fextr: vector of extremal frequencies

FIRPM is now a "function function", similar to CFIRPM, allowing you to write a function which defines the desired frequency response.

$B = \text{FIRPM}(N, F, @fresp, W)$  returns a length  $N+1$  FIR filter which has the best approximation to the desired frequency response as returned by the function handle  $@fresp$ . The function is called from within FIRPM using the syntax:

$$[DH, DW] = \text{fresp}(N, F, GF, W);$$

where:

$N$  is the filter order.

$F$  is the vector of frequency band edges which must appear monotonically between 0 and +1, where 1 is the Nyquist frequency. The frequency bands span  $F(k)$  to  $F(k+1)$  for  $k$  odd; the intervals  $F(k+1)$  to  $F(k+2)$  for  $k$  even are "transition bands" or "don't care" regions during optimization.

$GF$  is a vector of grid points which have been linearly interpolated over each specified frequency band by FIRPM, and determines the frequency grid at which the response function will be evaluated.

$W$  is a vector of real, positive weights, one per band, for use during optimization.  $W$  is optional; if not specified, it is set to unity weighting before being passed to 'fresp'.

$DH$  and  $DW$  are the desired complex frequency response and optimization weight vectors, respectively, evaluated at each frequency in grid  $GF$ .

The predefined frequency response function handle for FIRPM is  $@\text{firpmfrf}$ , but you can write your own. See the help for PRIVATE/FIRPMFRF for more information.

$B = \text{FIRPM}(N, F, \{ @fresp, P1, P2, \dots \}, W)$  specifies optional arguments  $P1$ ,  $P2$ , etc., to be passed to the response function handle  $@fresp$ .

$B = \text{FIRPM}(N, F, A, W)$  is a synonym for  $B = \text{FIRPM}(N, F, \{ @\text{firpmfrf}, A \}, W)$ , where  $A$  is a vector of response amplitudes at each band edge in  $F$ .

FIRPM normally designs symmetric (even) FIR filters.  $B = \text{FIRPM}(\dots, 'h')$  and  $B = \text{FIRPM}(\dots, 'd')$  design antisymmetric (odd) filters. Each frequency response function handle  $@fresp$  can tell FIRPM to design either an even or odd filter in the absence of the 'h' or 'd' flags. This is done

with:

```
SYM = fresp('defaults',{N,F,[],W,P1,P2,...})
```

FIRPM expects @fresp to return SYM = 'even' or SYM = 'odd'. If @fresp does not support this call, FIRPM assumes 'even' symmetry.

```
% Example of a length 31 lowpass filter:
```

```
h=firpm(30,[0 .1 .2 .5]*2,[1 1 0 0]);
```

```
% Example of a low-pass differentiator:
```

```
h=firpm(44,[0 .3 .4 1],[0 .2 0 0],'differentiator');
```

```
% Example of a type 4 highpass filter:
```

```
h=firpm(25,[0 .4 .5 1],[0 0 1 1],'h');
```

See also FIRPMORD, CFIRPM, FIRLS, FIR1, FIR2, BUTTER, CHEBY1, CHEBY2, ELLIP, FREQZ, FILTER, and, in the Filter Design Toolbox, FIRGR.