

CELL ASSEMBLY ENUMERATION IN RANDOM GRAPHS

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1 Introduction

1.1 History

Models of individual neurons vary in complexity, but in general, neurons tend to behave like this:

- A neuron is either excited or not excited.
- When excited, a neuron will stimulate neurons to which it has an outgoing connection. Otherwise, it will not stimulate other neurons.
- A neuron becomes excited when it receives a sufficient amount of stimulation from other neurons.

When modeling the brain, we seek to model the collective behavior of neurons. The fundamental type of collective behavior is, by the Donald Hebb model of the brain, the cell assembly.

First introduced by Hebb, the cell assembly is, "a diffuse structure comprising cells... capable of acting briefly as a closed system, delivering facilitation to other such systems and usually having a specific motor facilitation" [4, xix]. A cell assembly is a particular arrangement of a group of neurons with certain properties. The most salient of these properties is that a certain fractional portion of the assembly will excite the entire assembly.

By Hebb's proposal, a cell assembly represents a single concept in the brain. For instance, Hebb proposes that the corner of an abstract triangle may be represented by a cell assembly [4, 86]

Since Hebb first discussed the concept of a cell assembly, there has been some amount of biological research supporting his ideas. For instance, the work of György Buzsáki suggests that groups of cells that fire during a given time period are correlated [2, 160,161].

1.2 Definitions

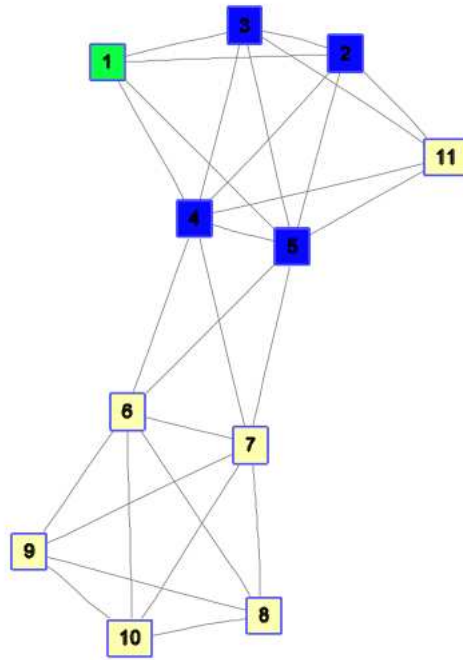


Figure 1: Node 1 (bright green) has the neighborhood $\{2, 3, 4, 5\}$ (dark blue)

Gunther Palm defined Hebb's assembly in the concrete language of graph theory [6]. In brief, Palm's discussion constructs, or depends upon, the following definitions:

- *graph*: A graph G has a vertex set, $V(G)$, and a set of edges, $E(G) \subseteq V(G) \times V(G)$. If $u, v \in V(G)$ and $uv \in E(G)$, we say that the graph G has an edge directed from the vertex u toward the vertex v . Vertices are labeled with integers by convention.
- *neighborhood*: We define the neighborhood of some vertex $v \in V(G)$ with respect to G , call it $N(v, G)$, as $\{u : \exists uv \in E(G)\}$ (figure 1).
- *degree*: The degree of a vertex $v \in V(G)$ with respect to G , call it $D(v, G)$, is $|N(v, G)|$.

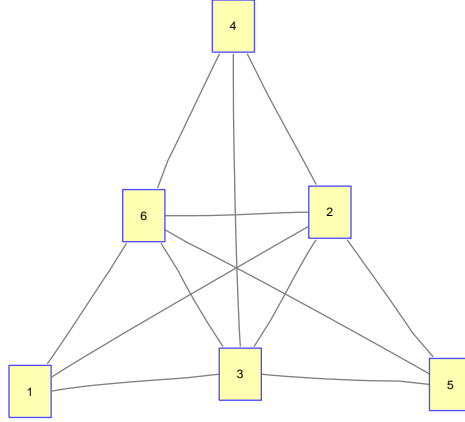


Figure 2: A k -core for which $k=3$

- *subgraph*: A graph g is a subgraph of G iff $V(g) \subseteq V(G)$ and $E(g) \subseteq E(G)$. Further, g is an induced subgraph of G iff g is a subgraph of G and $\forall e \in E(G) \cap (V(g) \times V(g)), e \in E(g)$.
- *k -core*: A subgraph g of G is a k -core iff $\forall v \in V(g), |X| \geq k$, where $X = N(v) \cap V(g)$ (figure 2).
- *minimal k -core* A subgraph g of G is a minimal k -core iff it has no induced subgraphs which are k -cores.
- *maximum k -core* A subgraph g of G is a maximum k -core iff G contains no k -cores h for which the $|V(h)| > |V(g)|$.
- *activation*: We say that a vertex $v \in V(G)$ can be either active or inactive. We generally say that, initially, an arbitrary subset of vertices $M \subseteq V(G)$ are active and the rest inactive. We further define a map $f_k(M, G) : (\text{sets of vertices, graphs}) \rightarrow (\text{sets of vertices})$ which performs the following operation:
 - Take a graph G , and a set of vertices, M , where $M \subseteq V(G)$.
 - $\forall v \in V(G)$:
 - * let $Y = M \cap N(v, G)$
 - * iff $|Y| \geq k$, then $v \in R$
 - Return R .

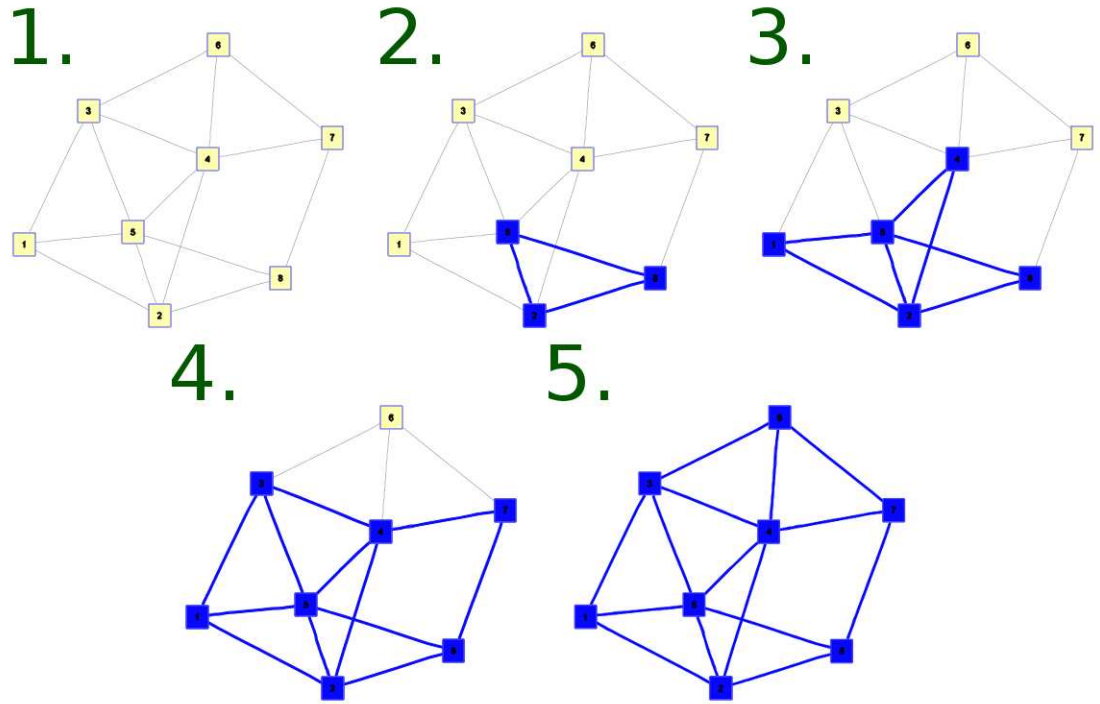


Figure 3: The process of closure for $k=2$: in step 2, an arbitrary 3 vertices are activated. Through each subsequent step, those vertices having at least 2 neighbors in the active set are activated in turn. After step 5, there are no more vertices to activate, so the vertices highlighted in step 5. are the closure of the vertices highlighted in step 2.

For convenience, we add a superscript $f_k^n(M, G)$, where $f_k^2(M, G) = f_k(f_k(M, G), G)$, $f_k^3(M, G) = f_k(f_k(f_k(M, G), G), G)$, etc.

- *closure*: We say that the closure of a set of active nodes M with respect to the graph G , call it $cl_k(M, G)$, is equal to $f_k^\infty(M, G)$. If $f_k^n(M, G)$ does not converge for some sufficiently large n , then the closure of M is undefined (figure 3).
- *k-tight*: A k -core T , which is an induced subgraph of G , is k -tight iff it satisfies the following condition:
 - $\forall K$ where K is an induced subgraph of T and a k -core:
 - * $cl(V(K), G) \supseteq V(T)$, or,
 - * $cl(V(T) \setminus V(K), G) = \emptyset$

- *k-assembly*: An induced subgraph A of G is a k -assembly iff $V(A) = cl(V(T), G)$ where T is a k -tight induced subgraph of G .

2 Goal

We seek to understand Palm's definition of cell assembly in context. In particular, we seek to determine whether a brain-sized random graph can contain a realistic number of assemblies by Palm's definition.

In this report, we describe the process of k-assembly enumeration and explain some preliminary experimentation using that algorithm.

The report exists in two forms. The abridged version includes the material described above, while the full version goes on to include more trend examples, an extensive collection of visual examples of k-Assemblies, and implementation.

3 Finding k-Assemblies

Finding k-assemblies takes place in two steps: k-core enumeration and k-assembly confirmation:

3.1 k-Core Enumeration

The process of k-Core enumeration follows the general form of a branch and bound algorithm. That is, it follows the process:

1. If possible, solve the problem, otherwise:
2. Break the problem into several smaller problems, for each of these smaller problems, go to step 1.

Applied to our problem, the general algorithm looks like this, given some input graph G (figure 4):

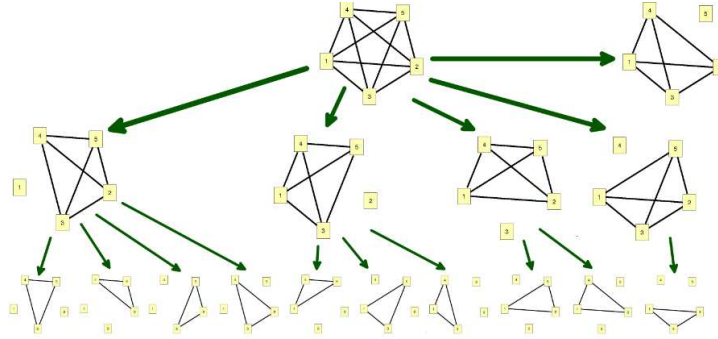


Figure 4: An exhaustive 2-core enumeration: Ultimately, every induced subgraph is enumerated except those that are not 2-cores.

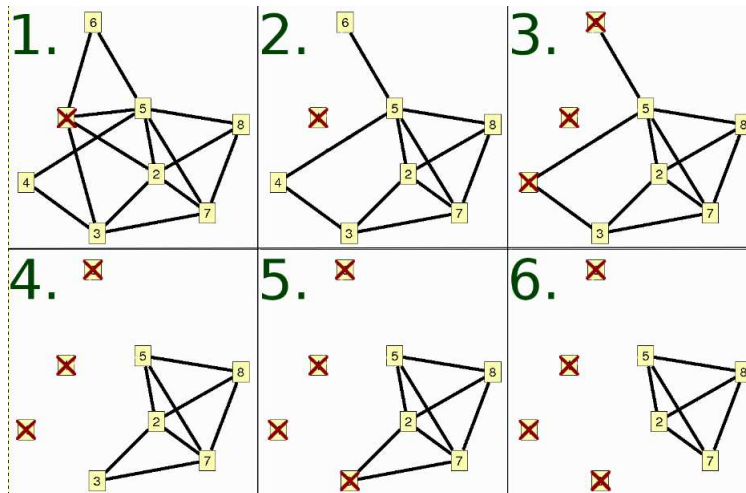


Figure 5: The maximum k -core algorithm for a 3-core: 1,2) A vertex is eliminated from the graph, leaving an induced subgraph which is not a minimal 3-core. 3,4) All vertices which have insufficient degree with respect to the new subgraph (less than 3 in this case) are eliminated. 5,6) Again, a vertex without sufficient degree is eliminated. The subgraph pictured in step 6 has no vertices of less than k degree, so the process is completed.

1. If G is a minimal k -core, stop, otherwise:
2. Dissect G as follows:

- (a) Find the maximum k -core that is an induced subgraph of G , call it H (figure 5).
- (b) For each vertex $v \in V(H)$, go to step 1, using for the new G and induced subgraph of H such that $V(G) = V(H) \setminus v$.

This process finds all k -cores in the original G . To sketch a proof:

- An algorithm that finds all induced subgraphs in G and then filters k -cores from the rest will trivially enumerate all induced k -cores in G .
- The k -core enumeration algorithm described is equivalent to such an algorithm. By in turn eliminating every vertex from a graph, the algorithm find all induced subgraphs except for those it skips. The skipped subgraphs will never generate k -cores not already generated:
 - Skipped subgraphs have at least one vertex of degree less than k , with respect to that subgraph; call this vertex set W_1 . The subgraph may contain vertices that would have degrees less than k if excluding the vertices in W_1 ; call this set W_2 . There may also be vertices dependent on the vertices in W_1 and W_2 to maintain a degree of k ; call those W_3 . We can continue forming these sets until $G \setminus (W_1 \cup W_2 \cup \dots \cup W_n)$ is a k -core for some positive integer n . Define W as $(W_1 \cup W_2 \cup \dots \cup W_n)$
 - Trivially, take some induced subgraph of G called g . $\forall v \in V(g)$, $D(v, g) \leq D(v, G)$
 - Consequently, no induced subgraph of G will contain a k -core including any vertices in W , since no vertex in W has gained degree upon finding an induced subgraph, and, consequently, the collective W still cannot meet the degree threshold to be included in a k -core.
 - Subgraphs are skipped iff they contain vertices in W .

3.1.1 Complexity

The algorithm described above runs in approximately exponential ($O(n!)$) time, which is to be expected since it solves an NP-hard problem.

The enumeration of k -cores reduces to the NP complete clique problem [5, 618-619] as follows:

- A clique is a k -core for which the number of vertices in the core is equal to $k + 1$ (figure 6).
- The clique problem is: determine whether an arbitrary graph contains a clique of at least size n .
- If we could accomplish k -core enumeration in polynomial time, say $O(n^k)$, where n is the number of nodes in the graph, then:
 - We could find cores of all k in, at most, $O(n \times n^k) = O(n^{k+1})$ time, because there is no k core in a given graph for which k exceeds n .

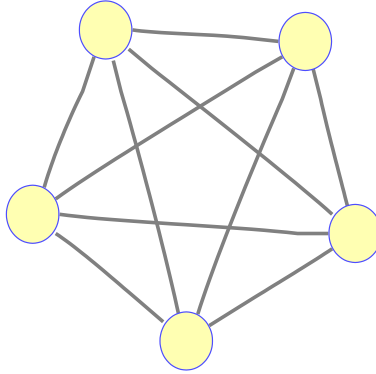


Figure 6: A clique for which $k = 4$

- Without increasing fundamental run-time, we could flag each of those k -cores which is a clique. In doing so, we find all cliques.
- By simply finding the largest of this group of cliques, we have solved the clique problem in polynomial time.

Consequently, k -core enumeration belongs to the class of NP-hard problems, meaning that it is not clear whether an algorithm that runs significantly faster than exponential time can be devised.

That is not to say, however, that the algorithm cannot be improved upon at all. For instance, one of our implementations takes advantage of the fact that, if we form an induced subgraph H by removing some vertex v from a graph G , if v is not in a k -core, then the graph I resulting from removing any of v 's neighbors that are not in a k -core from G will have the same maximum k -core as H . Also, our algorithm takes care not to revisit previously enumerated branches. Different approaches can certainly speed up an algorithm to solve the problem, but it is not clear that any approach will make the algorithm run in sub-exponential time.

3.2 Assembly Enumeration

Upon enumerating all k -cores, every unique closure of a k -core is analyzed to determine whether any one of the k -cores that closes to that closure is tight (figure 7). Checking tightness involves nothing more clever than simply verifying the above definition of tight. If at least one of the cores that closes to a given closure is tight, then that closure is collected into the set of k -assemblies, otherwise, it is ignored.

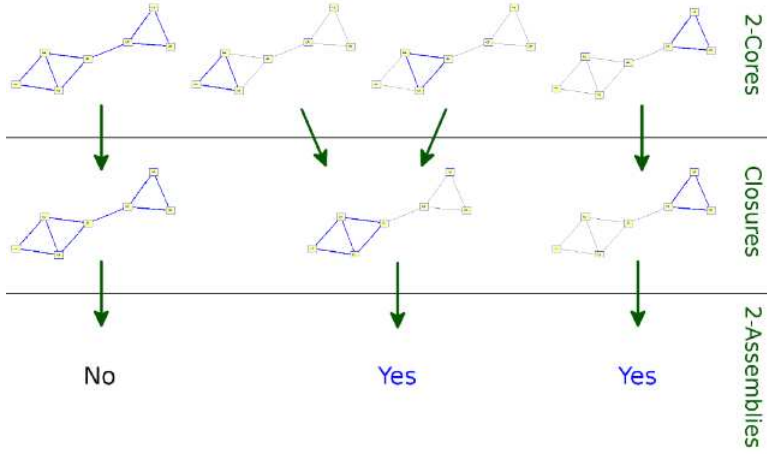


Figure 7: k-Assembly enumeration on a reduced set of k-cores.

4 Experimentation

After devising an algorithm to find every k-assembly, we attempted to discover attributes about k-assemblies through, first, generating random graphs, and then, enumerating the assemblies contained in those graphs.

4.1 Random Graph Generation

We employed two primary types of random graphs.

1. The classical Bernoulli random graph (figure 4.1):
 - Pick some probability p , and a number of vertices, n . Create a graph G , for which $V(G) = 1, 2, \dots, n$.
 - For all pairs of vertices, (i, j) , where $i \neq j$ and $0 < i, j \leq n$, \exists an edge $ij \in E(G)$ with probability p .
2. The scale-free Cooper-Frieze random graph (figure 4.1) [1, 14-15]. As we implement it:

- pick some positive integer T ; $\alpha, \beta, \gamma, \delta \in [0, 1]$; P, Q , which are 1-indexed lists.

$$P = (p_1, p_2, \dots, p_n), \text{ where } \sum_{i=1}^n p_i = 1, \text{ and } p_i \in [0, 1] \forall i \in \{1, 2, \dots, n\}.$$

$$Q = (q_1, q_2, \dots, q_m), \text{ where } \sum_{i=1}^m q_i = 1, \text{ and } q_i \in [0, 1] \forall i \in \{1, 2, \dots, m\}.$$

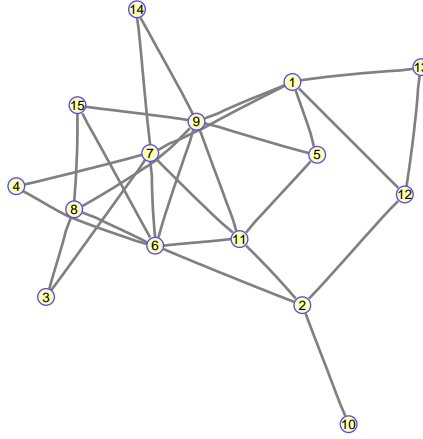


Figure 8: A Bernoulli random graph

- Begin with a graph G , where $V(G) = \{1\}$, and $E(G) = \{11\}$. (That is, G is a graph with a single vertex which has a single edge connected to itself).
- $\forall t \in \{0, 1, 2, \dots, T\}$:
 - Do the "Old" procedure with probability α , otherwise, do the procedure "New."
 - Do the procedure "Add Edges."
- *Old:*
 - with probability δ , choose the vertex *start* from among the set of vertices in $V(G)$ randomly, giving each vertex an even chance. Otherwise, choose *start* with the probability for each vertex proportional to the the degree of that vertex with respect to G .
 - with probability γ set the boolean variable *terminateUniformly* to *true*. Otherwise, set the variable to *false*.
 - choose an index of Q so that the index i has a probability q_i of being chosen. Set the integer variable *numberOfEdges* to this chosen index.
- *New:*
 - Add a new vertex v to $V(G)$. Call that vertex *start*.
 - With probability β set the boolean variable *terminateUniformly* to *true*. Otherwise, set the variable to *false*.

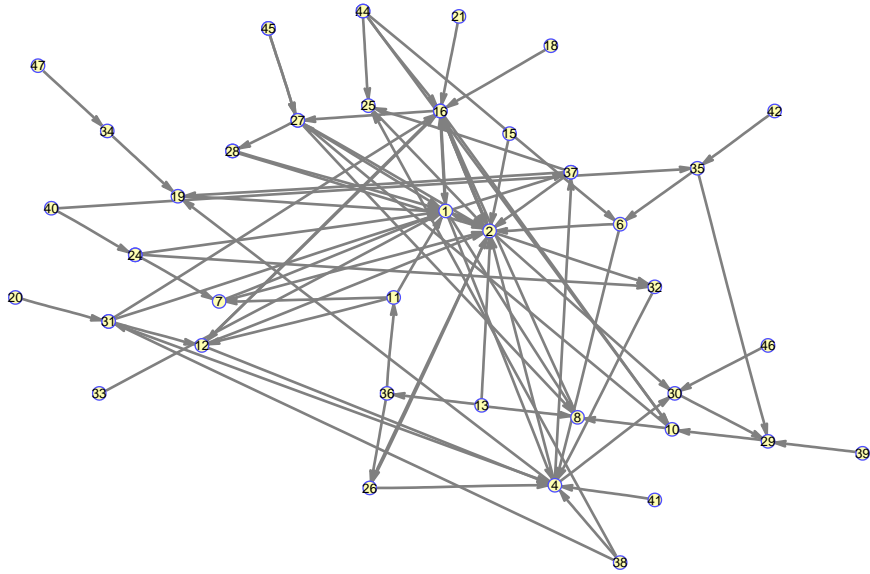


Figure 9: A scale-free Cooper-Frieze random graph

– choose an index of P so that the index i has a probability p_i of being chosen. Set the integer variable *numberOfEdges* to this chosen index.

• *Add Edges:*

– Create the set *END* by choosing *numberOfEdges* vertices from G . The vertices are chosen randomly, either with uniform probability if *terminateUniformly* is *true*, or in proportion to degree otherwise.

– \forall vertices $e \in END$, add to G an edge directed from *start* to e .

It should be noted that throughout the book, only scale-free graphs are allowed either:

- Edges with weights greater than 1 (That is, there may exist more than one edge $uv \in E(G)$ where u and v are vertices in $V(G)$.)
- "Loops" which connect nodes to themselves (That is, edges of the form uu .)

We allow these exceptions so that our graphs will be in full compliance with the Cooper-Frieze model.

4.2 Data

After generating a number of random graphs, we investigated a number of relationships between cell assemblies vs. other features in random graphs. We speculate on a few of these relationships here.

When we vary the probability that any given pair of vertices has an edge between them over a number of undirected, Bernoulli random graphs, we can see a number of distinct phases in the number of cell assemblies that these graphs will have, on average (figure 10).

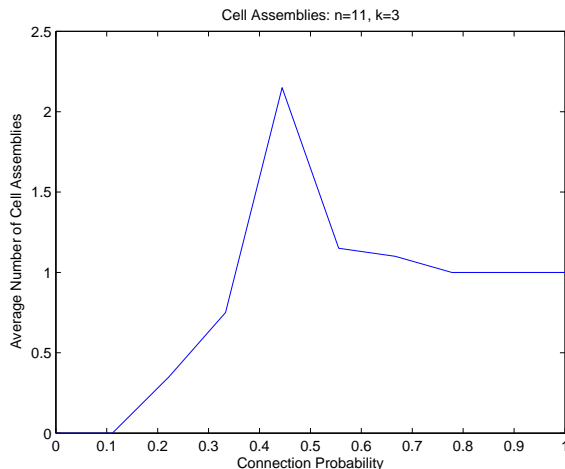


Figure 10: Bernoulli random graphs appear to undergo phase changes when varying edge probability

First, edge probability is insufficient to form even one assembly, then the number of assemblies increases to a peak as the graph becomes more dense. Past that peak, the closure of all tight sets converges to a single cell assembly. It would be interesting to find a method for predicting the probability at which this peak occurs, and then comparing that value to biology.

The Cooper-Frieze scale-free graph has no parameter to directly adjust edge density, but varying the related parameter, α , does have similar effects.

A fairly promising, although ostensibly not very accurate, indicator of the number of k -cores is the maximum eigenvalue of a graph's adjacency matrix (figure 11).

On undirected Bernoulli random graphs, the two variables appear to have a nontrivial, positive correlation. However, here we see that the method of graph construction is extremely important to any such observations, since on directed scale-free graphs, we see a relationship that is not nearly as clear (figure 12).

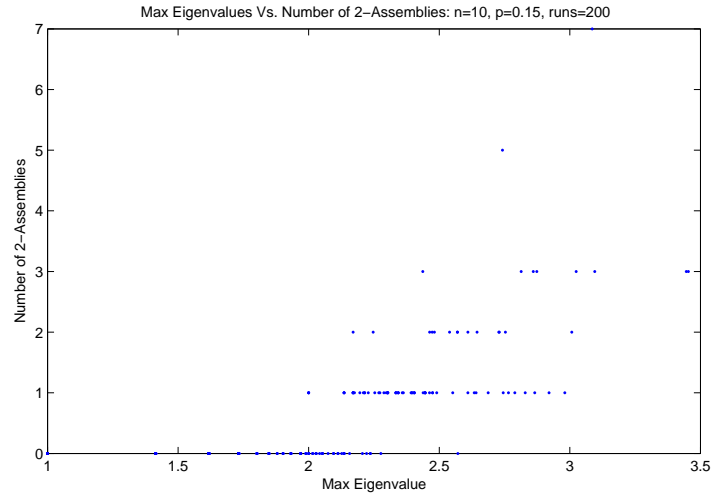


Figure 11: Maximum eigenvalues apparently have a positive correlation with number of assemblies in a Bernoulli random graph. ($r = 0.7309$)

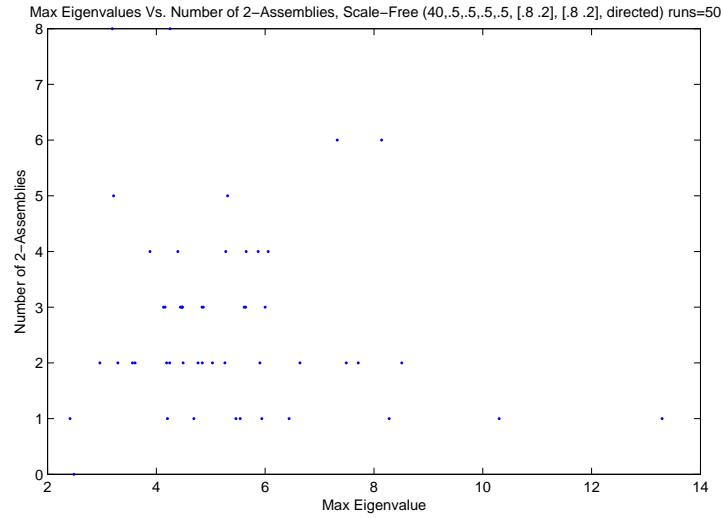


Figure 12: For Cooper-Frieze random graphs, this correlation is much more dubious. ($r = -0.1315$)

5 Future Work

- On commodity hardware, the k-core enumeration algorithm terminates in a few minutes on graphs on the order of 10 nodes. It can process larger graphs given a low density of edges or a large k. Consequently, it is not clear that strict enumeration will be useful on human-brain-sized graphs with approximately 100 billion neurons and 1000 connections per neuron, but it may be useful for smaller graphs. For instance, a honey bee has fewer than 1 million neurons [3].
- If strict enumeration fails even in smaller cases, it would perhaps be possible to take advantage of certain knowledge about the structure of brains in order to speed up enumeration by restricting the types of graphs that need to be enumerated. For instance, if we could safely use a bipartite model for a brain, the algorithm could likely be optimized for bipartite graphs.
- Also, cell assembly enumeration could proceed further from a statistical standpoint, as hinted at in the "Data" section. Perhaps some function could be devised that maps some more easily discerned information about a graph to the probability that that graph will contain a given number of cell assemblies.
- The definition of cell assemblies, as presented here, is fairly cumbersome. If certain aspects of the definition, such as closure, could be simplified, perhaps an easier approach to cell assemblies may become apparent.
- Cell assemblies should theoretically have far more function than simply existing as static structures in random graphs. Studying the interaction of cell assemblies and the learning processes that create cell assemblies may yield interesting insights into neuroscience in general.

6 Conclusion

Our approach to enumeration of cell assemblies in arbitrary graphs probably runs insufficiently fast to explore the problem as an end in itself. However, it does give us a useful tool to help us understand cell assemblies. We can now find an unlimited number of examples of cell assemblies which we may use as tools to explore general trends and gain insight into the structure of cell assemblies. Perhaps with enough work on the subject, we may find a viable way to understand the workings of the brain through random graph theory.

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A Trend Visualization

All Bernoulli random graphs in this section are undirected.

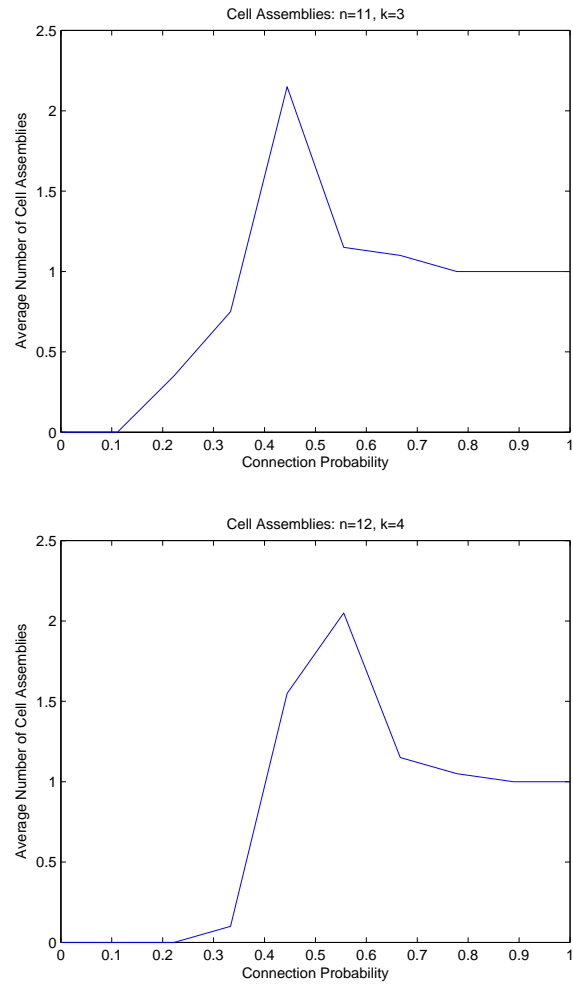
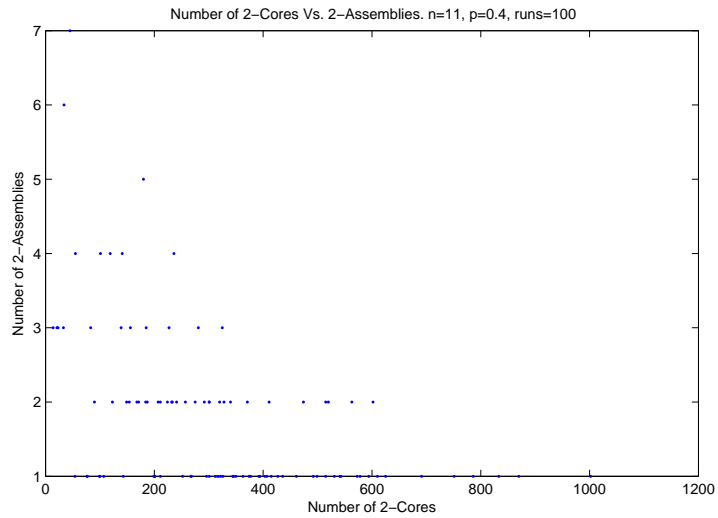
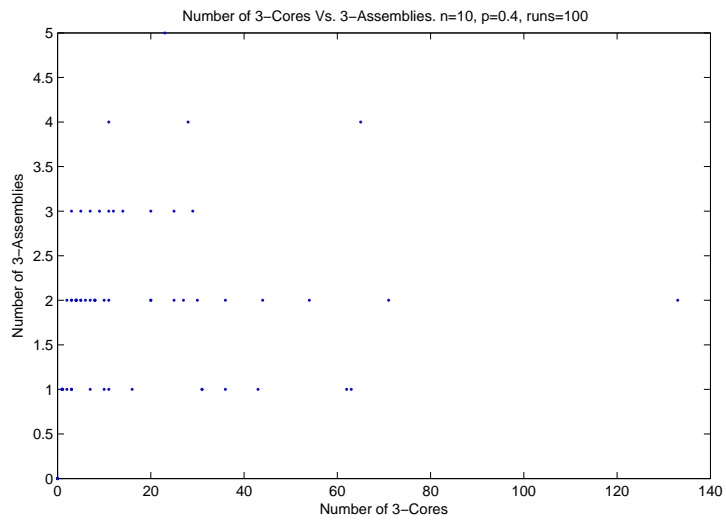


Figure 13: A comparison of number of assemblies vs. Edge Probability over the average of 10 Bernoulli random graphs per time-step

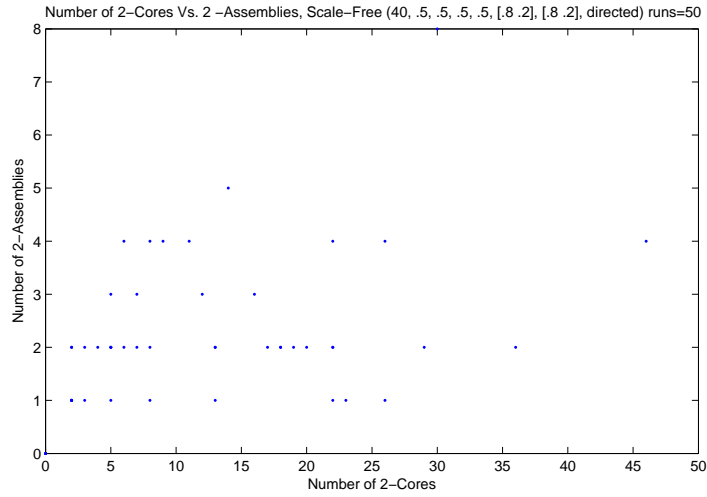


(a) $r = -0.4858$

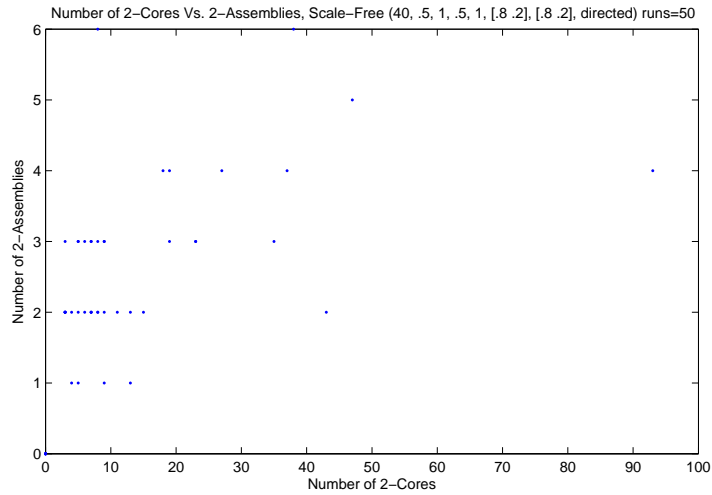


(b) $r = 0.3781$

Figure 14: A comparison of number of cores vs. assemblies over 100 Bernoulli random graphs

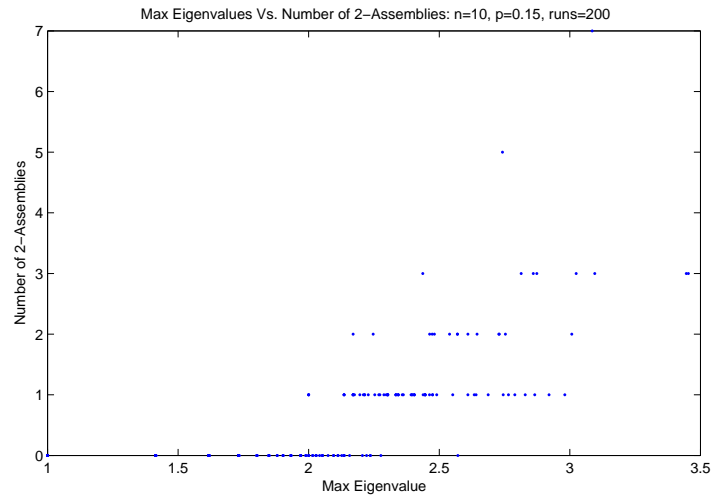


(a) $r = 0.4323$

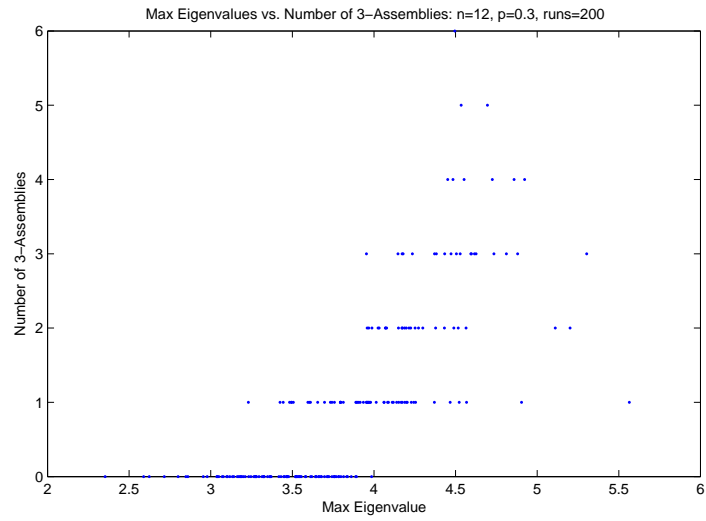


(b) $r = 0.5510$

Figure 15: A comparison of number of cores vs. assemblies over 100 Cooper-Frieze random graphs

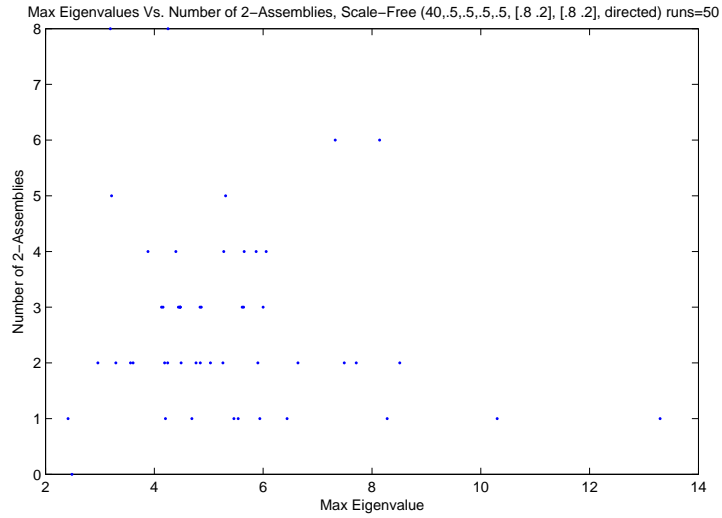


(a) $r = 0.7309$

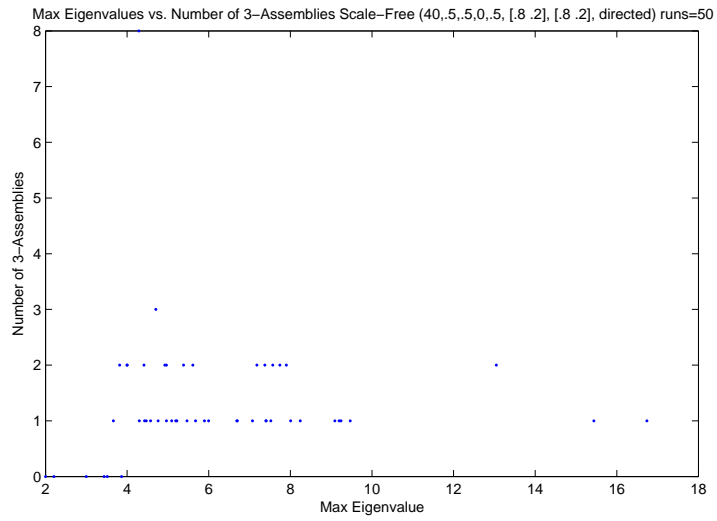


(b) $r = 0.7495$

Figure 16: A comparison of number of cores vs. eigenvalues in 200 Bernoulli random graphs



(a) $r = -0.1315$



(b) $r = 0.0175$

Figure 17: A comparison of number of cores vs. eigenvalues in 50 Cooper-Frieze random graphs

B Examples

B.1 A note on random graph generation

Examples labeled with the prefix "randomGraph" are Bernoulli random graphs. Their labels follow the format:

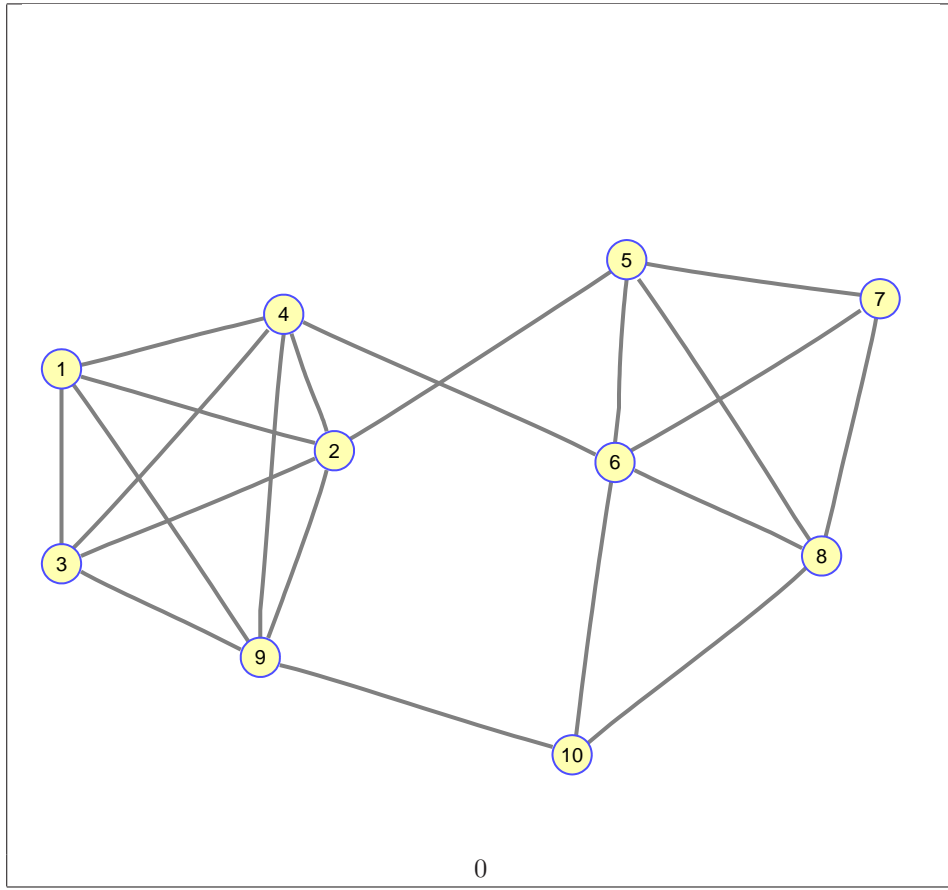
randomGraph(number of nodes, edge density, [un]directed)-unique id

Examples labeled with the prefix scaleFreeGraph are Cooper-Frieze random graphs. Their labels follow the format:

scaleFreeGraph(T, alpha, beta, gamma, delta, p, q, [un]directed)-unique id
(where p and q are both vectors whose entries add up to 1.)

B.2 Undirected Graphs

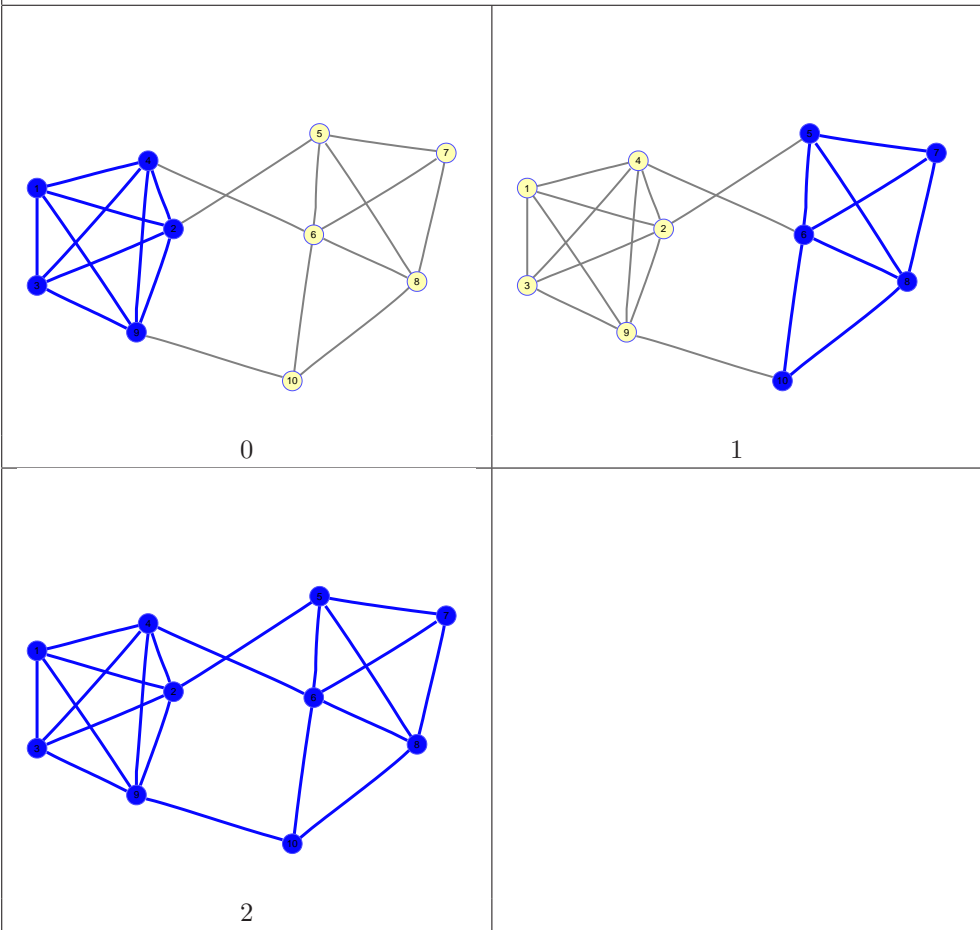
B.2.1 adj01



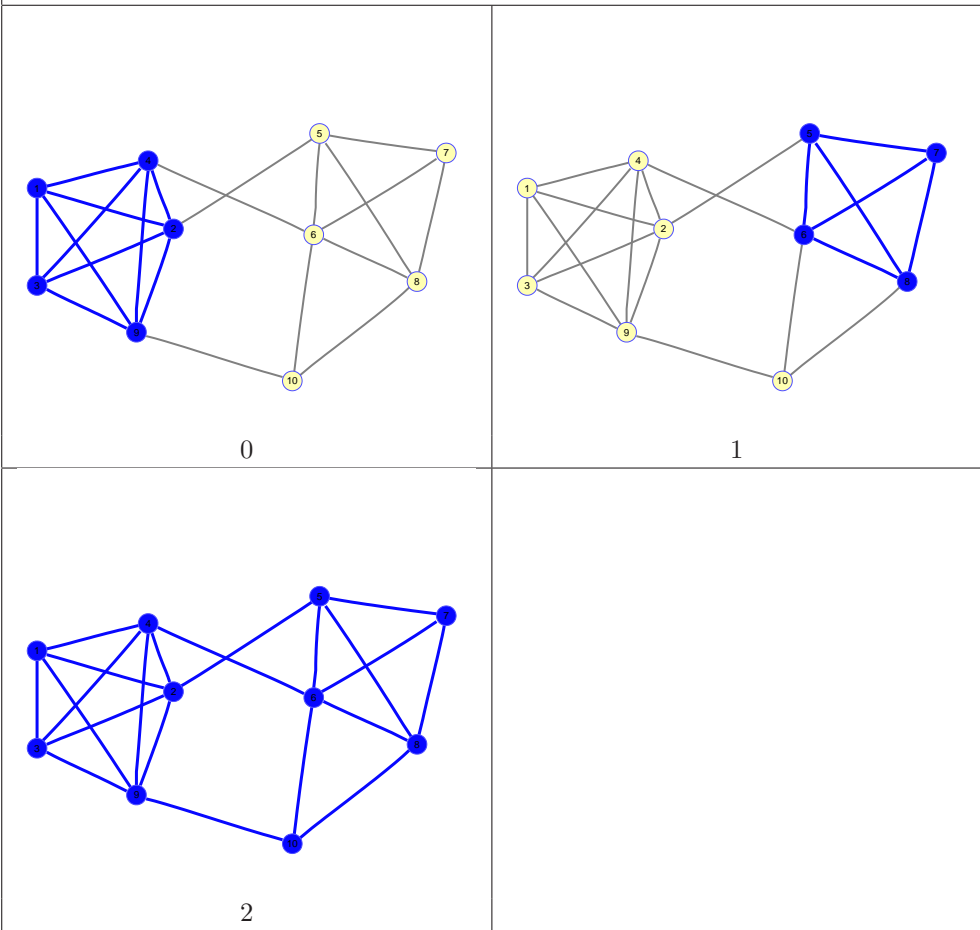
0	1	1	1	0	0	0	0	1	0
1	0	1	1	1	0	0	0	1	0
1	1	0	1	0	0	0	0	1	0
1	1	1	0	0	1	0	0	1	0
0	1	0	0	0	1	1	1	0	0
0	0	0	1	1	0	1	1	0	1
0	0	0	0	1	1	0	1	0	0
0	0	0	0	1	1	1	0	0	1
1	1	1	1	0	0	0	0	0	1
0	0	0	0	0	1	0	1	1	0

(1)

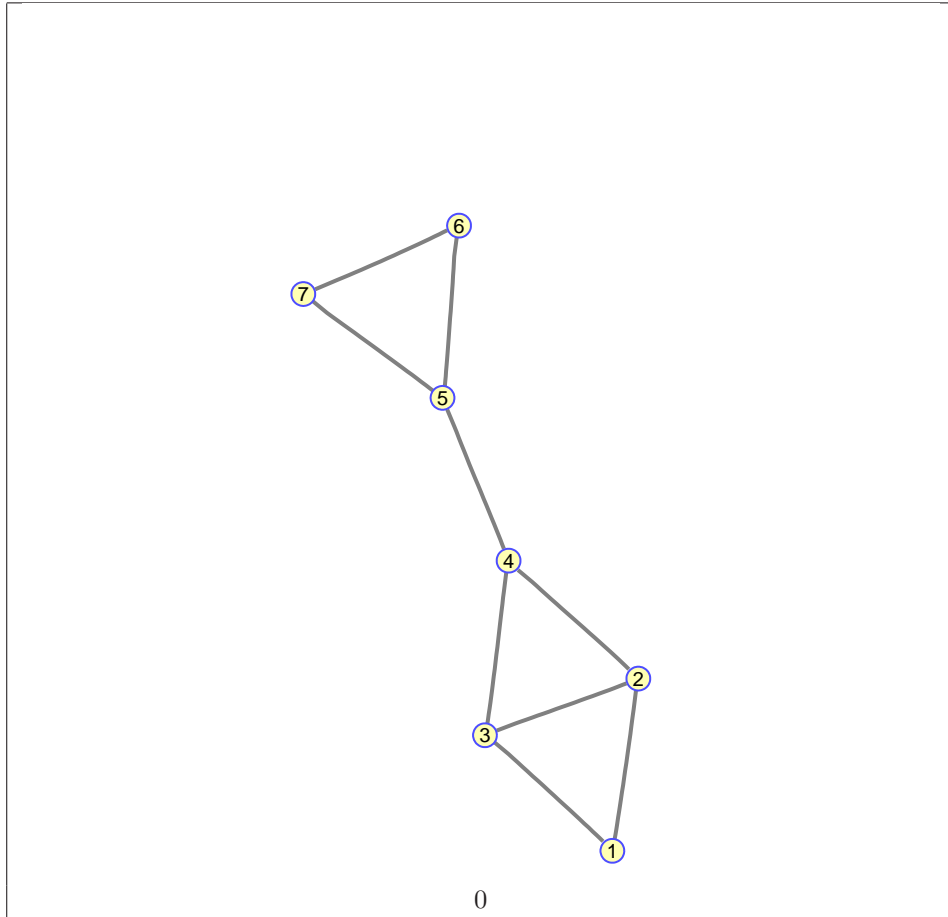
2-Assemblies



3-Assemblies



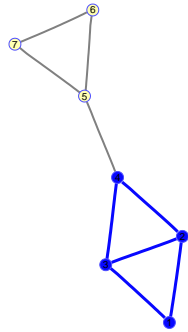
B.2.2 adj02



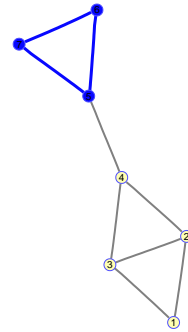
```
0 1 1 0 0 0 0
1 0 1 1 0 0 0
1 1 0 1 0 0 0
0 1 1 0 1 0 0
0 0 0 1 0 1 1
0 0 0 0 1 0 1
0 0 0 0 1 1 0
```

(2)

2-Assemblies

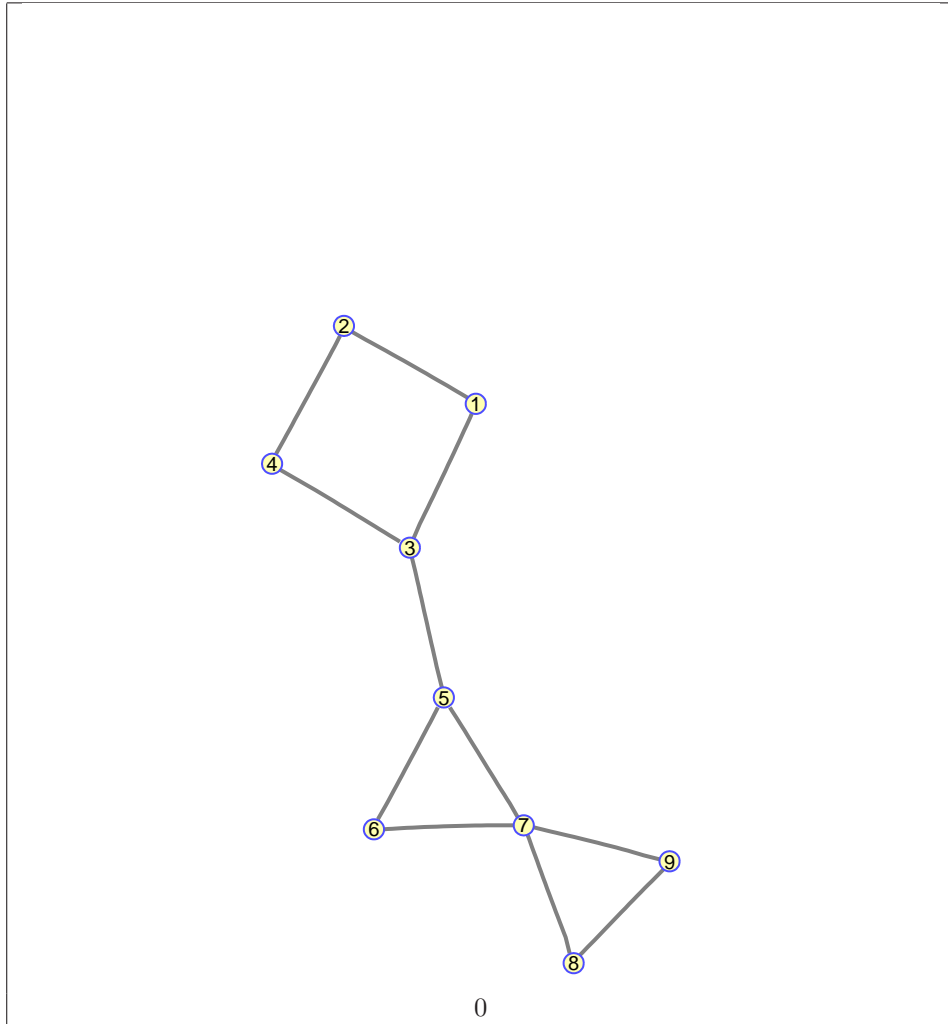


0



1

B.2.3 adj03



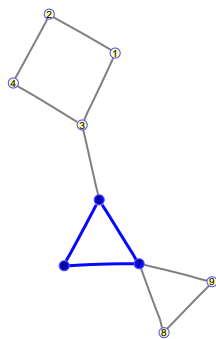
```

0 1 1 0 0 0 0 0 0 0
1 0 0 1 0 0 0 0 0 0
1 0 0 1 1 0 0 0 0 0
0 1 1 0 0 0 0 0 0 0
0 0 1 0 0 1 1 0 0 0
0 0 0 0 1 0 1 0 0 0
0 0 0 0 1 1 0 1 1 1
0 0 0 0 0 0 1 0 1 1
0 0 0 0 0 0 1 1 0
0 0 0 0 0 0 1 1 0

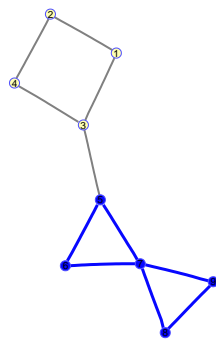
```

(3)

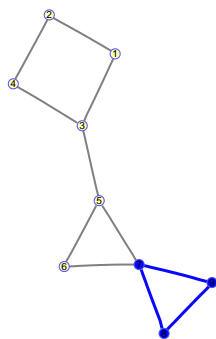
2-Assemblies



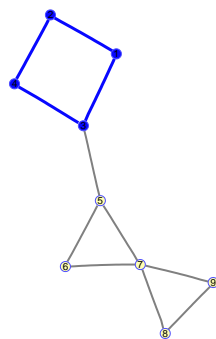
0



1

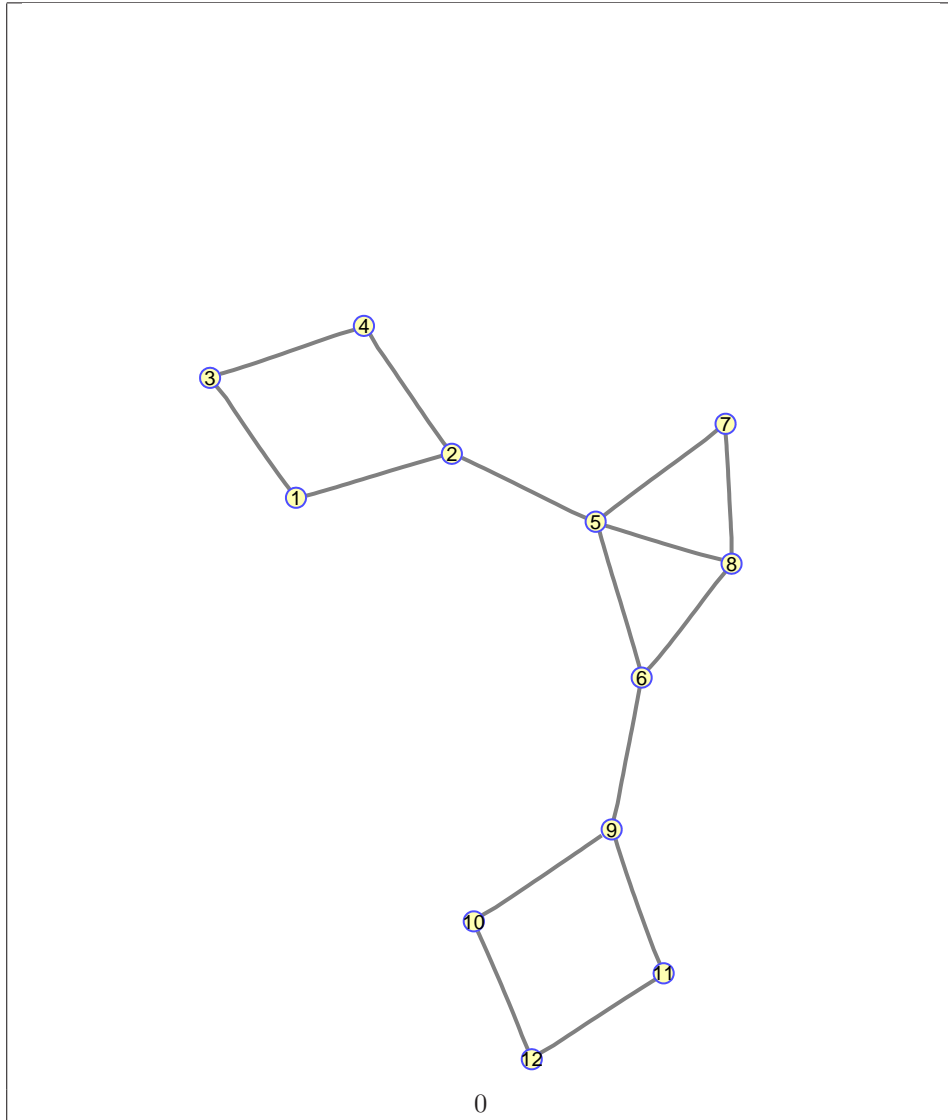


2



3

B.2.4 adj04



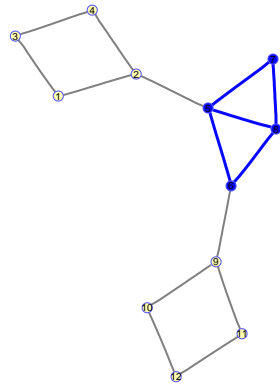
```

0 1 1 0 0 0 0 0 0 0 0 0
1 0 0 1 1 0 0 0 0 0 0 0
1 0 0 1 0 0 0 0 0 0 0 0
0 1 1 0 0 0 0 0 0 0 0 0
0 1 0 0 0 1 1 1 0 0 0 0
0 0 0 0 1 0 0 1 1 0 0 0
0 0 0 0 1 0 0 1 0 0 0 0
0 0 0 0 1 1 1 0 0 0 0 0
0 0 0 0 0 1 0 0 0 1 1 0
0 0 0 0 0 0 0 0 1 0 0 1
0 0 0 0 0 0 0 0 1 0 0 1
0 0 0 0 0 0 0 0 0 1 1 0

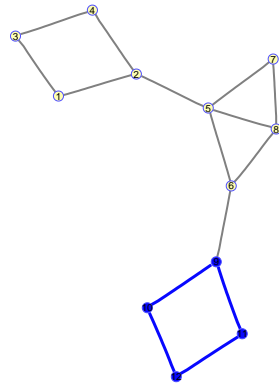
```

(4)

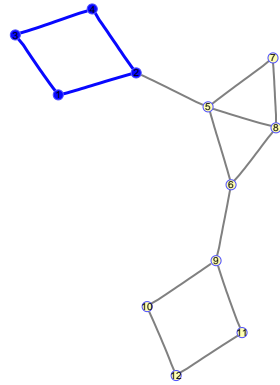
2-Assemblies



0

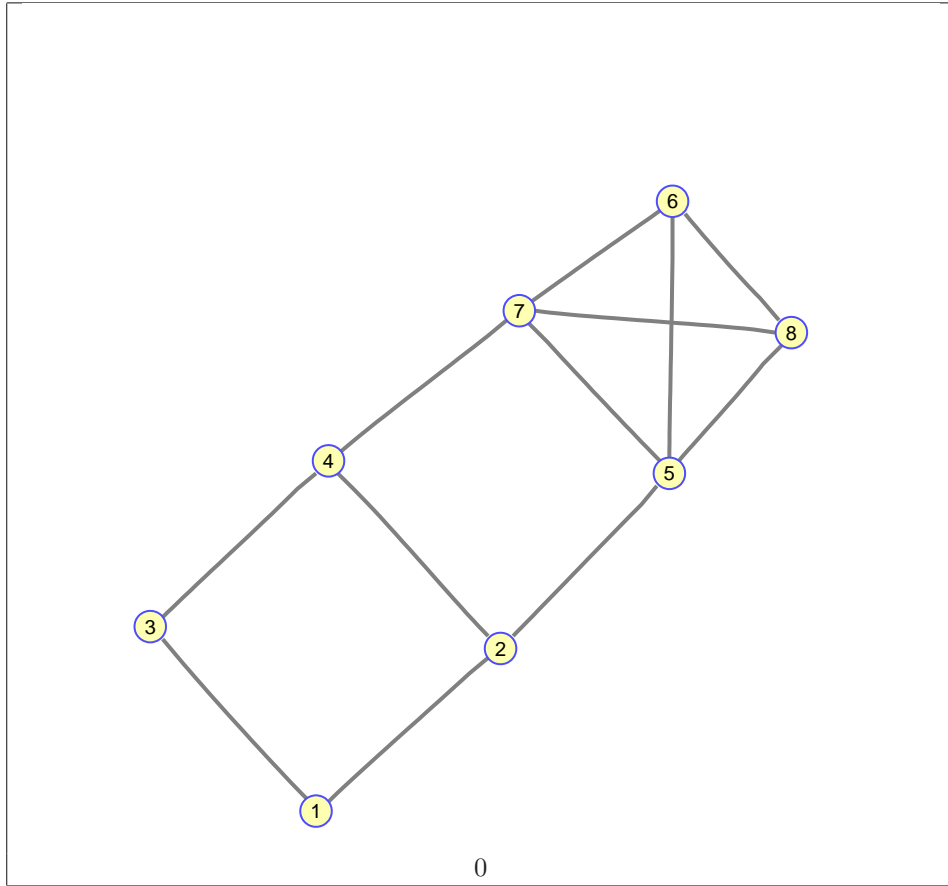


1



2

B.2.5 adj05

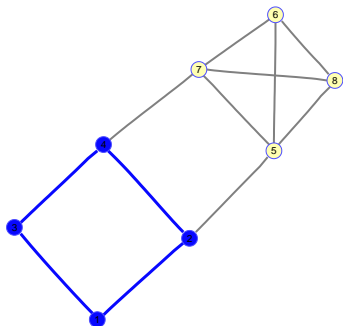


0

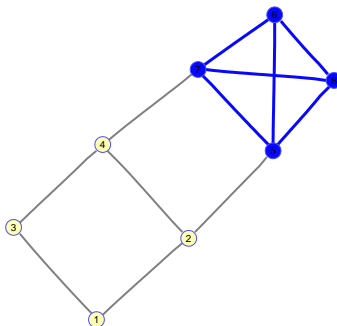
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1	0	0	1	1	0	0	0
1	0	0	1	0	0	0	0
0	1	1	0	0	0	1	0
0	1	0	0	0	1	1	1
0	0	0	0	1	0	1	1
0	0	0	1	1	1	0	1
0	0	0	0	1	1	1	0

(5)

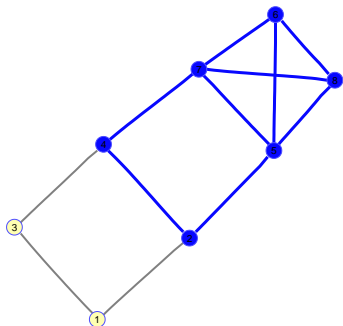
2-Assemblies



0

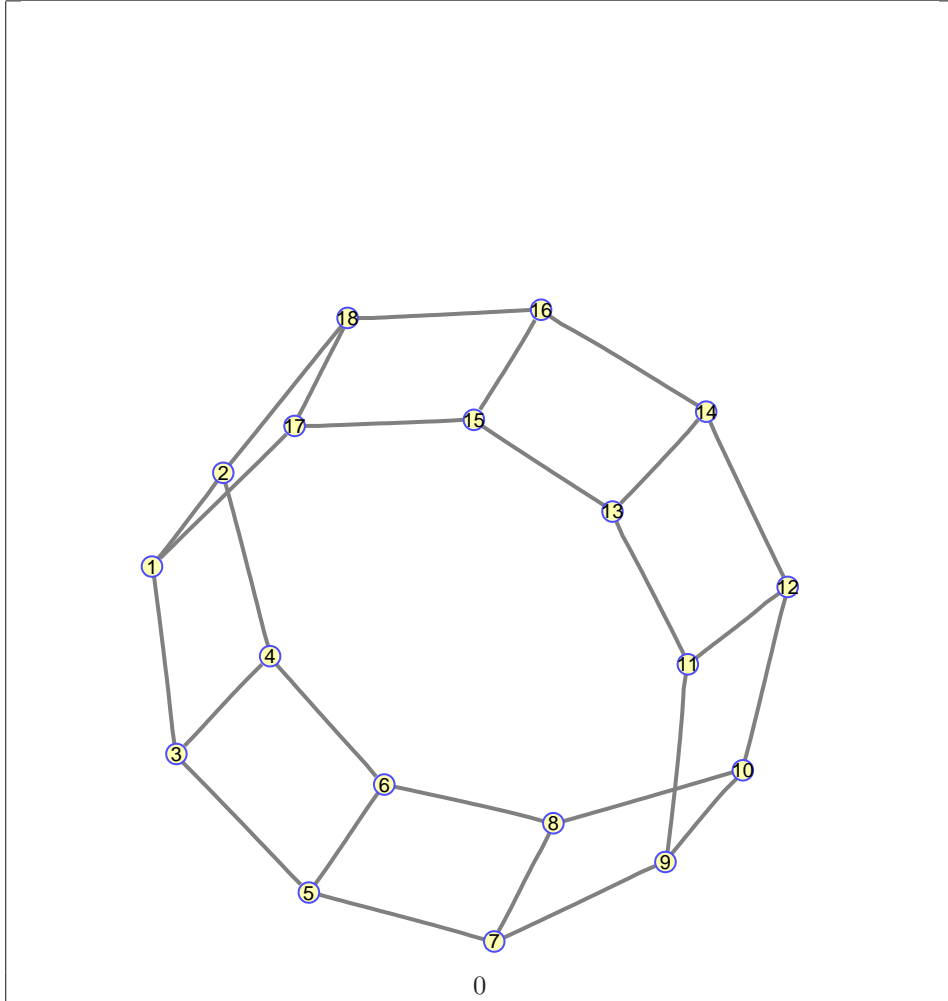


1



2

B.2.6 adj06

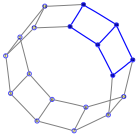
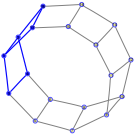
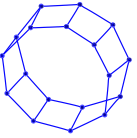
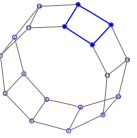
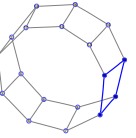
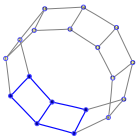
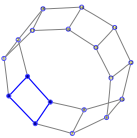
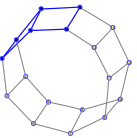
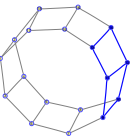
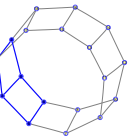
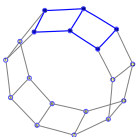
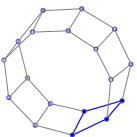
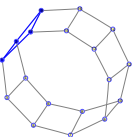
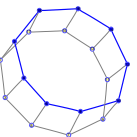
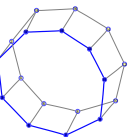
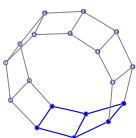
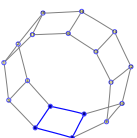
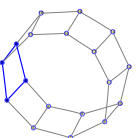
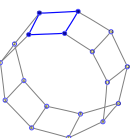
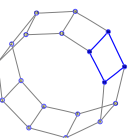
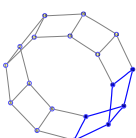


```

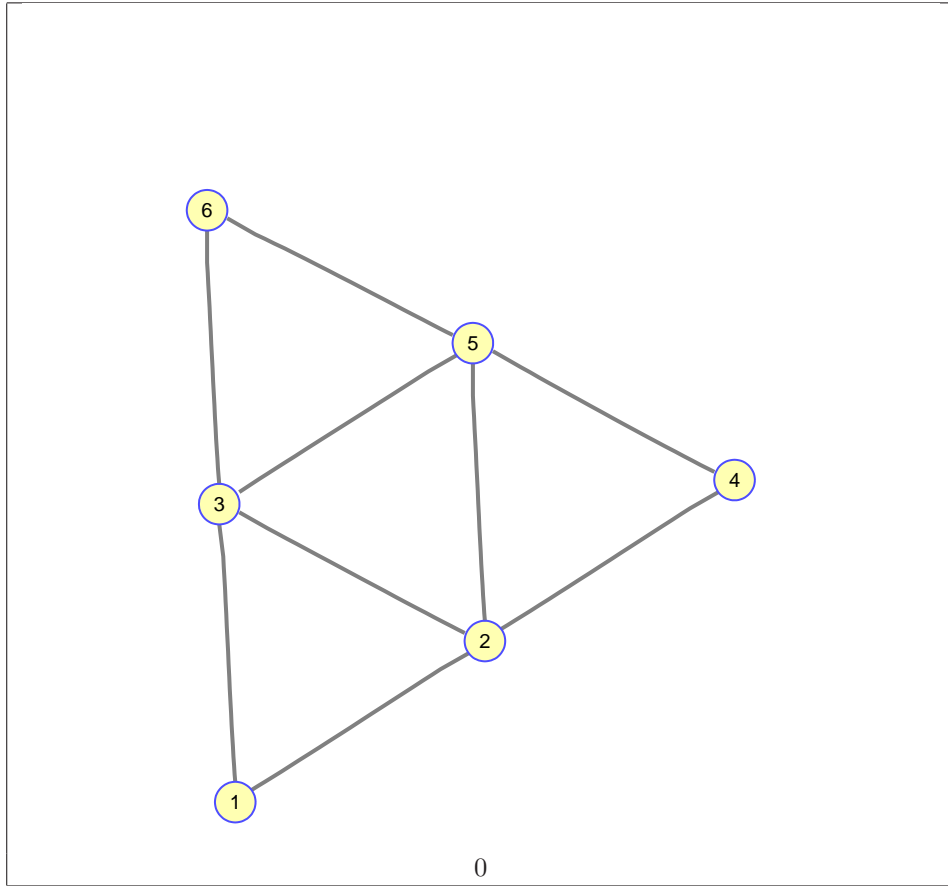
0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1
1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0

```

(6)

2-Assemblies				
 0	 1	 2	 3	 4
 5	 6	 7	 8	 9
 10	 11	 12	 13	 14
 15	 16	 17	 18	 19
 20				

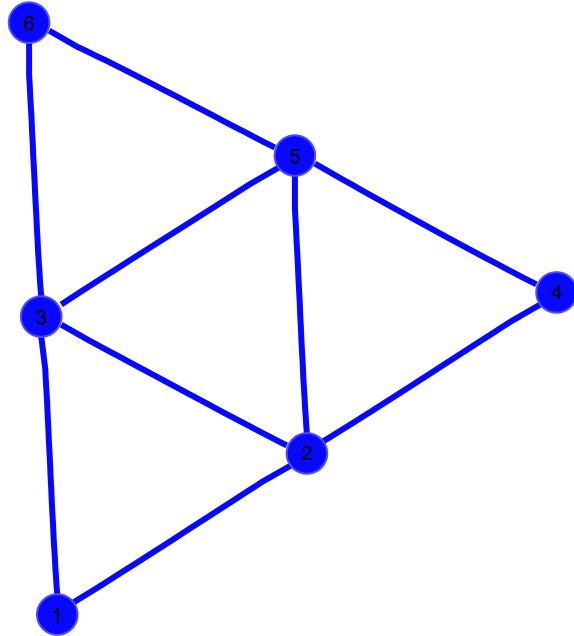
B.2.7 adj08



0 1 1 0 0 0
1 0 1 1 1 0
1 1 0 0 1 1
0 1 0 0 1 0
0 1 1 1 0 1
0 0 1 0 1 0

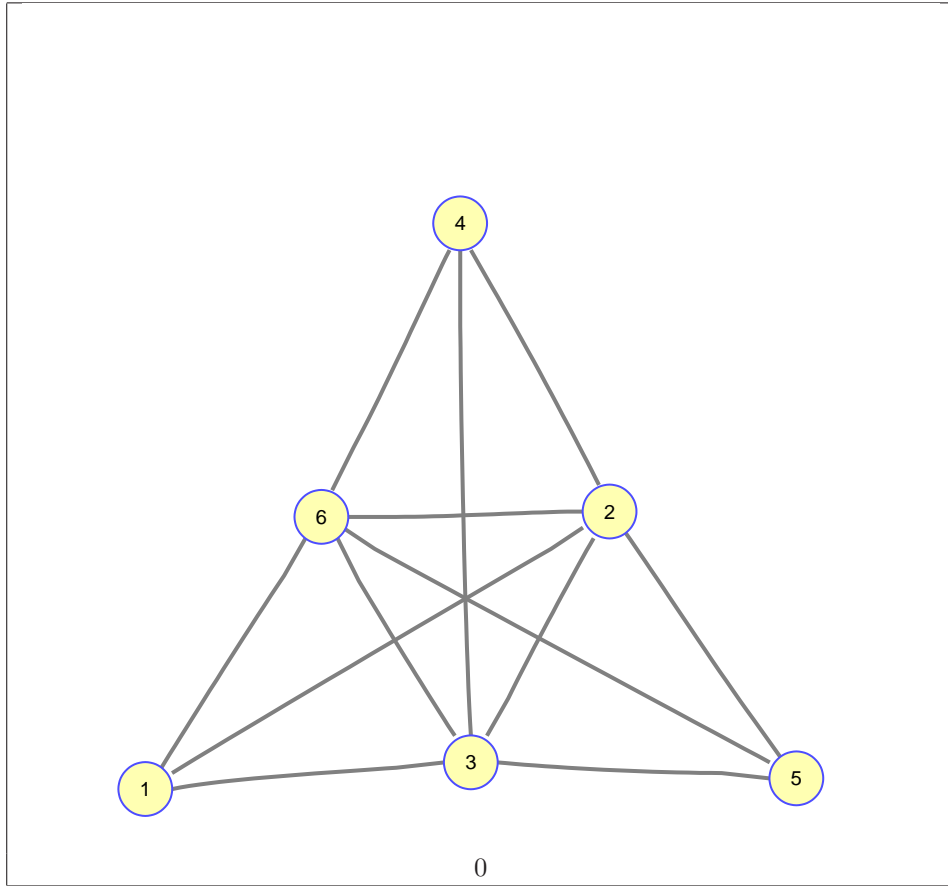
(7)

2-Assemblies



0

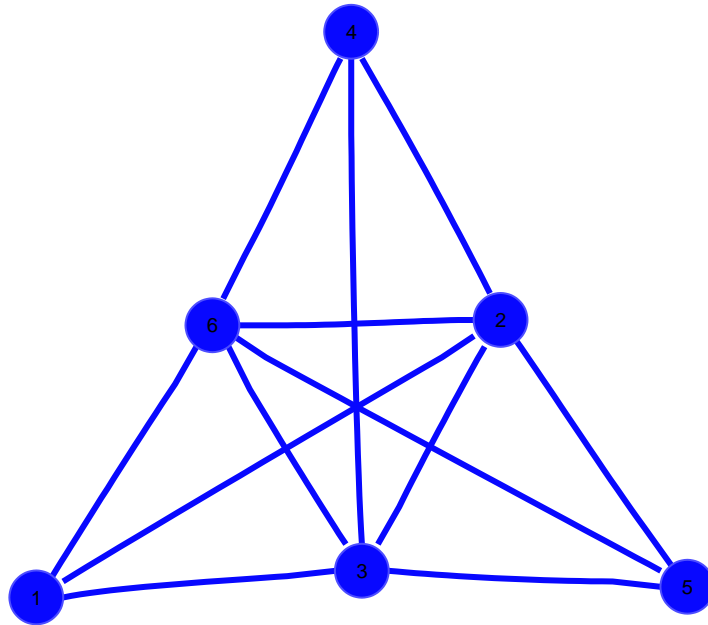
B.2.8 adj09



0 1 1 0 0 1
1 0 1 1 1 1
1 1 0 1 1 1
0 1 1 0 0 1
0 1 1 0 0 1
1 1 1 1 1 0

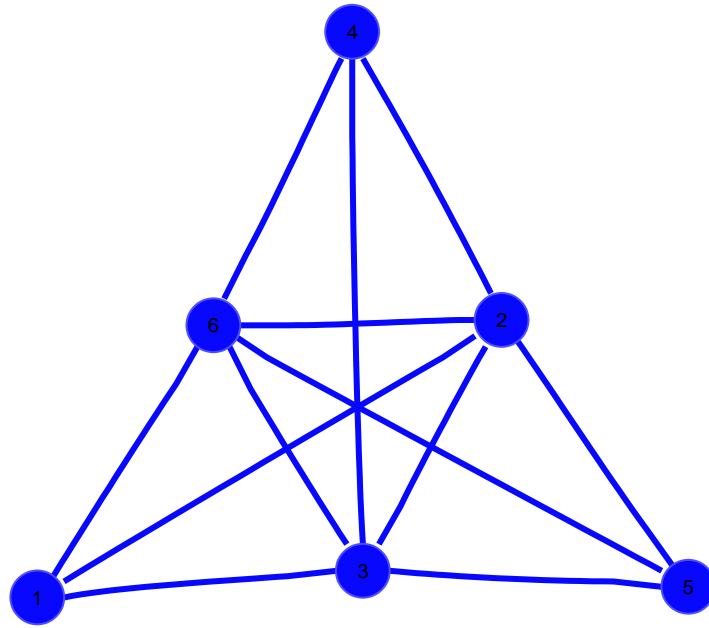
(8)

2-Assemblies



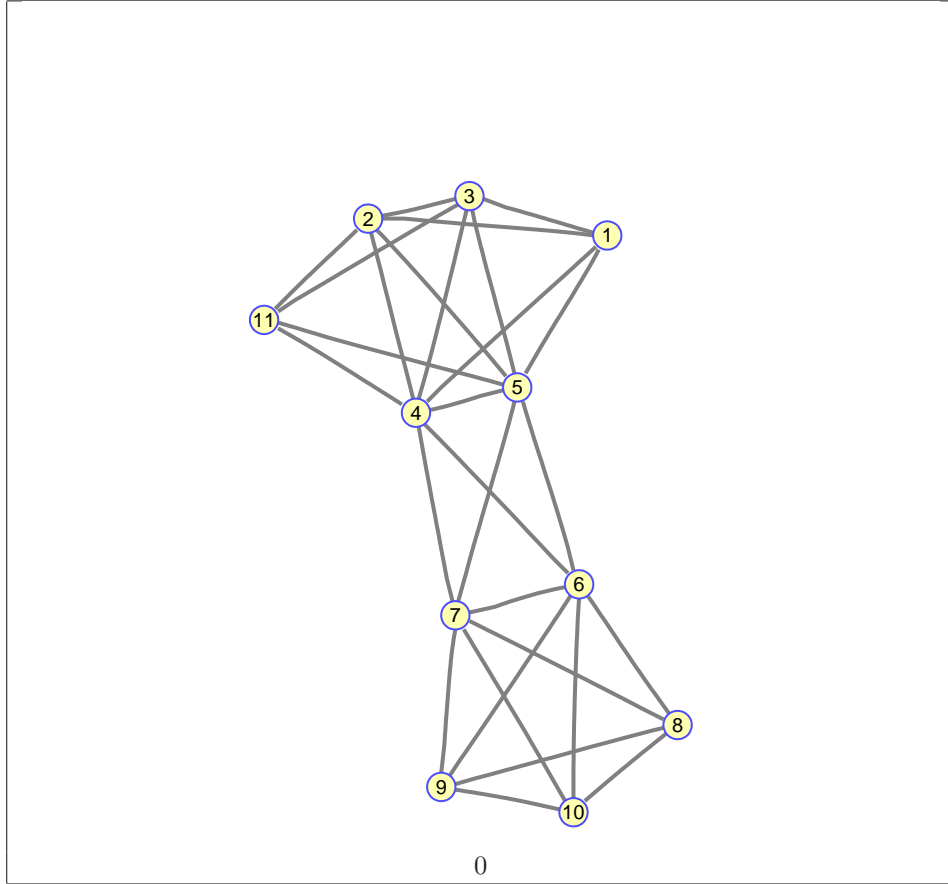
0

3-Assemblies



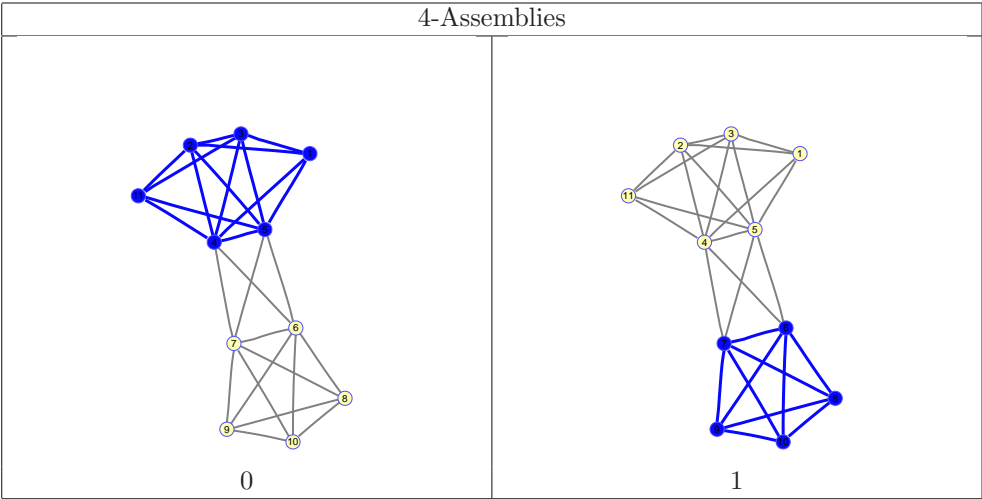
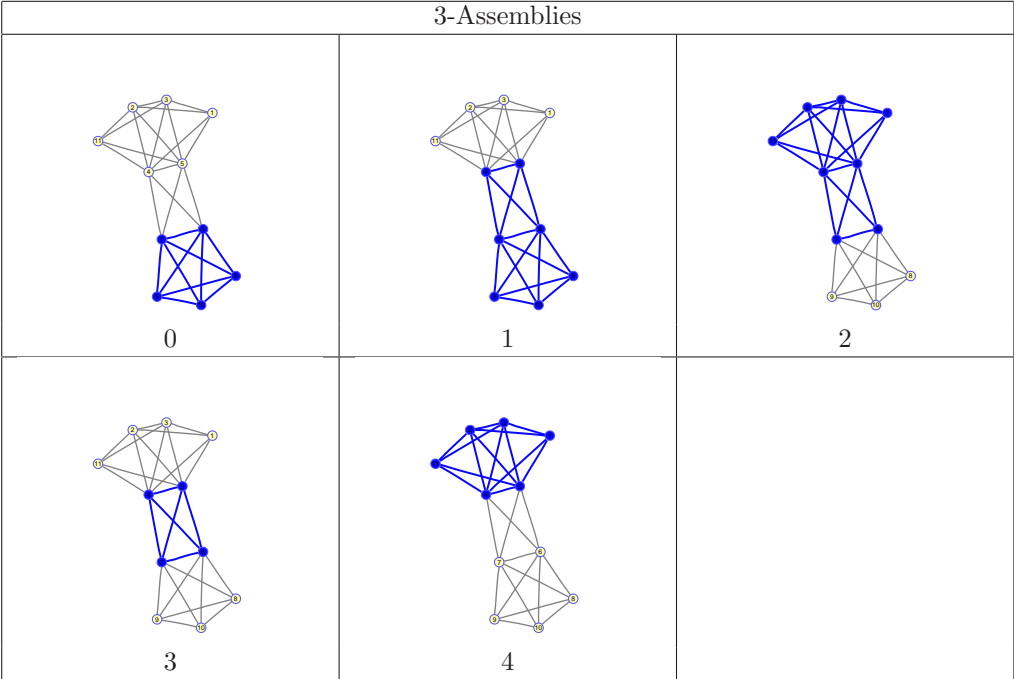
0

B.2.9 adj10

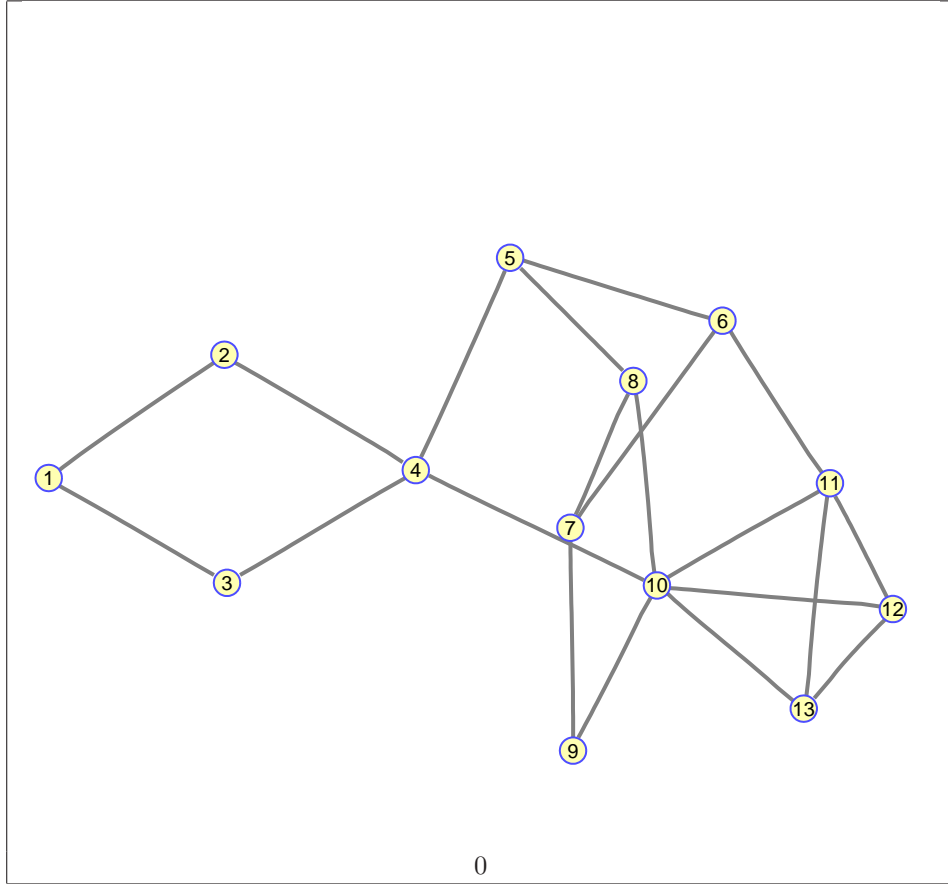


0	1	1	1	1	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0	1
1	1	0	1	1	0	0	0	0	0	1
1	1	1	0	1	1	1	0	0	0	1
1	1	1	1	0	1	1	0	0	0	1
0	0	0	1	1	0	1	1	1	1	0
0	0	0	1	1	1	0	1	1	1	0
0	0	0	0	0	1	1	0	1	1	0
0	0	0	0	0	1	1	1	0	1	0
0	0	0	0	0	1	1	1	1	0	0
0	1	1	1	1	0	0	0	0	0	0

(9)



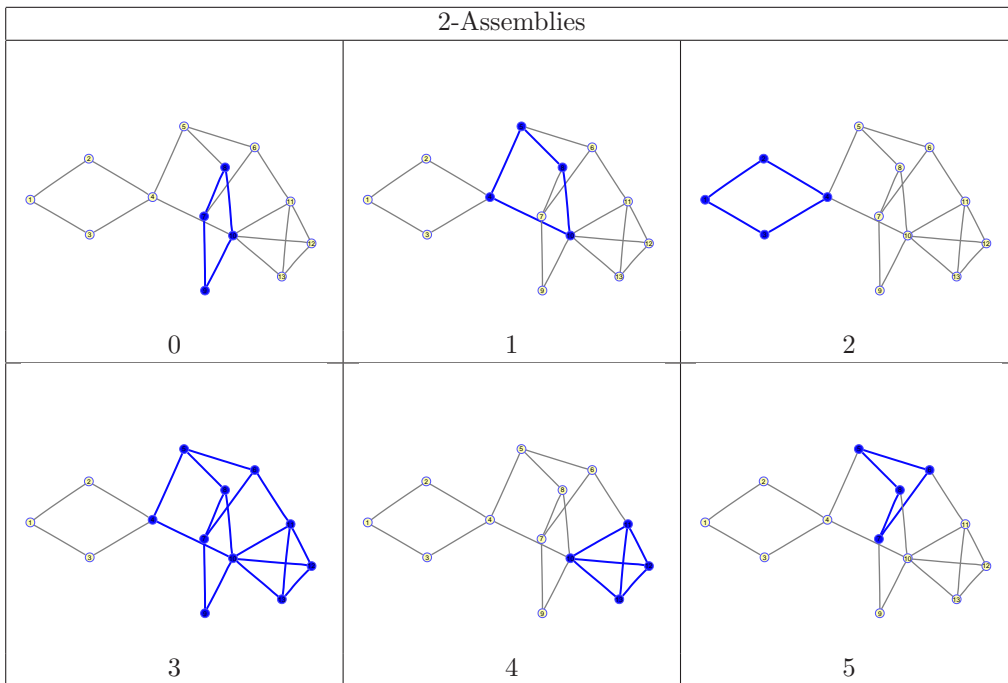
B.2.10 adj11



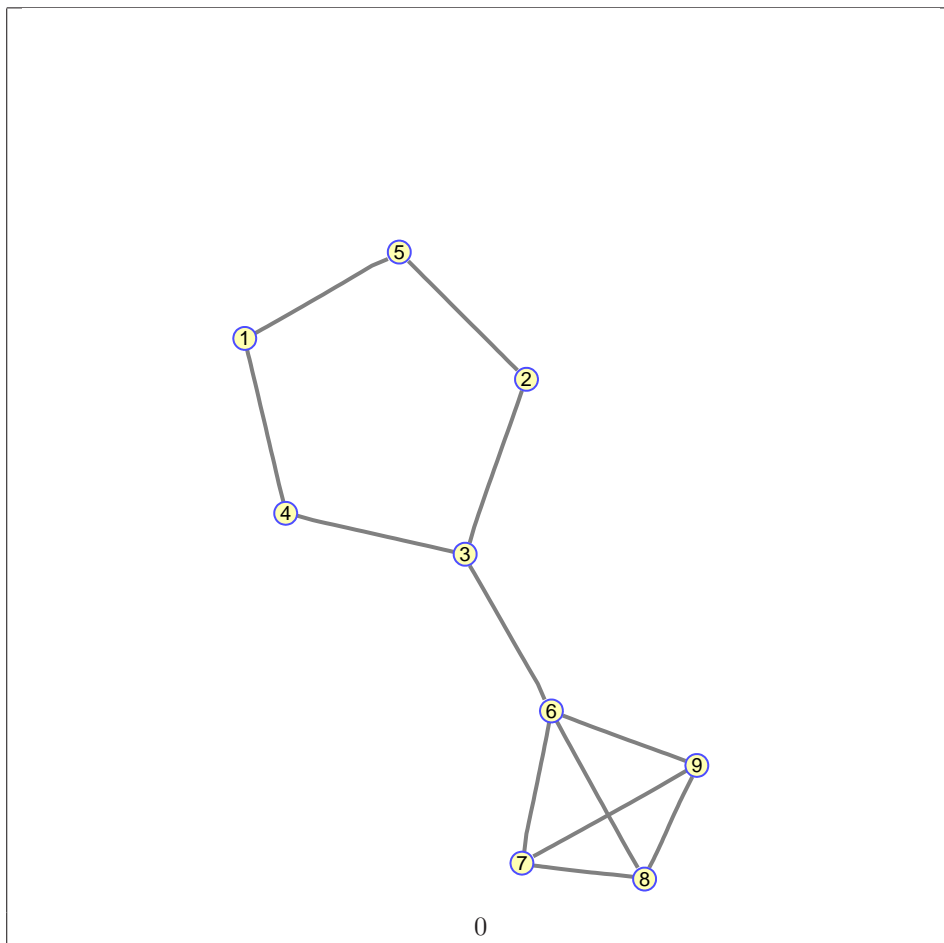
0	1	1	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0	0	0
0	1	1	0	1	0	0	0	0	1	0	0	0
0	0	0	1	0	1	0	1	0	0	0	0	0
0	0	0	0	1	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	1	1	0	0	0	0
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0	0	0	0	0	1	0	0	0	1	0	1	1
0	0	0	0	0	0	0	0	0	1	1	0	1
0	0	0	0	0	0	0	0	0	1	1	1	0

(10)

2-Assemblies



B.2.11 adj12



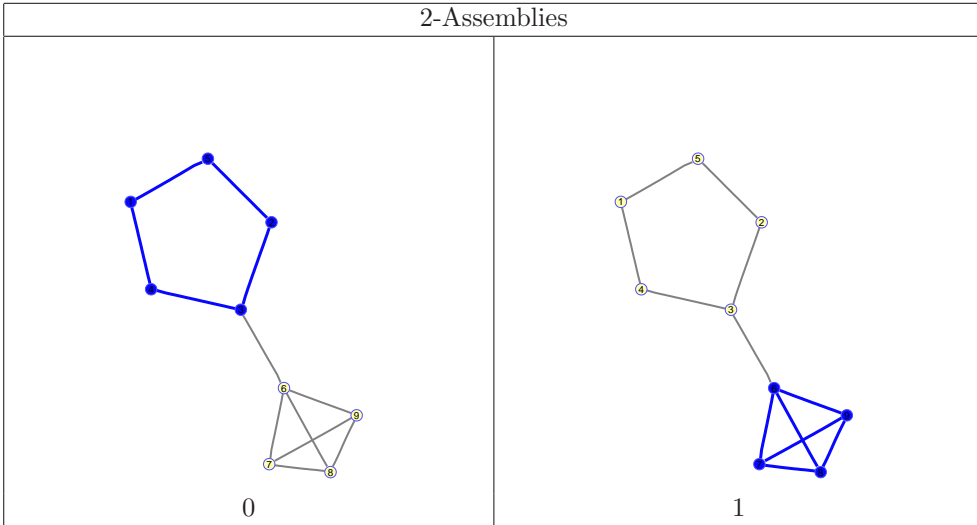
```

0 0 0 1 1 0 0 0 0
0 0 1 0 1 0 0 0 0
0 1 0 1 0 1 0 0 0
1 0 1 0 0 0 0 0 0
1 1 0 0 0 0 0 0 0
0 0 1 0 0 0 1 1 1
0 0 0 0 0 1 0 1 1
0 0 0 0 0 1 1 0 1
0 0 0 0 0 1 1 1 0

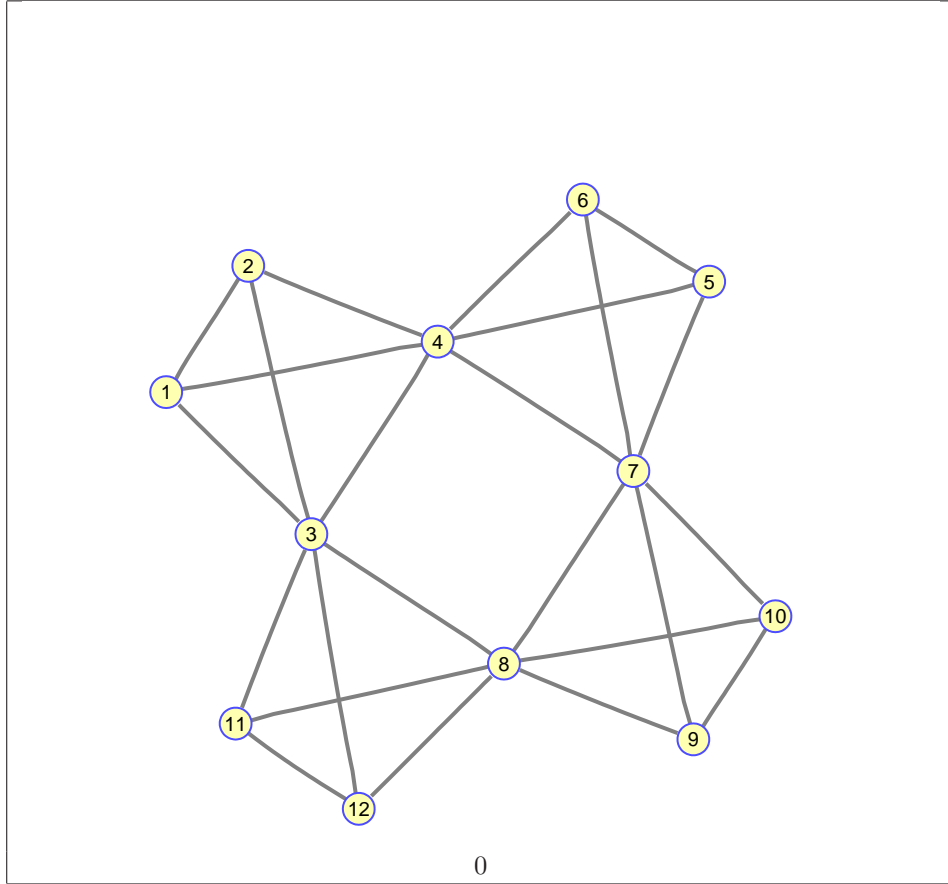
```

(11)

2-Assemblies

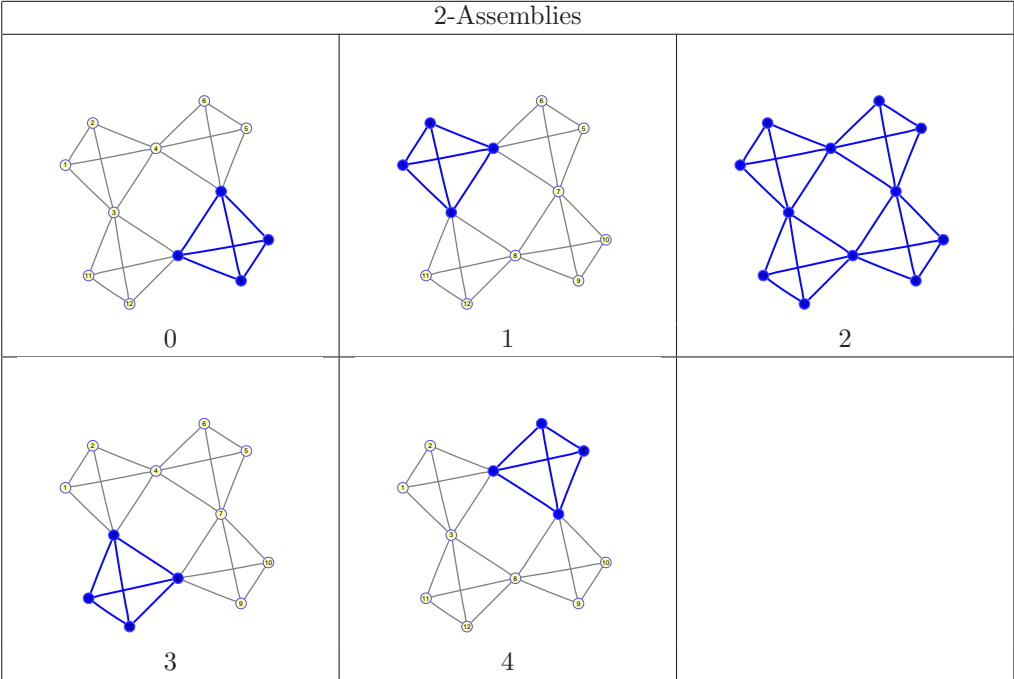


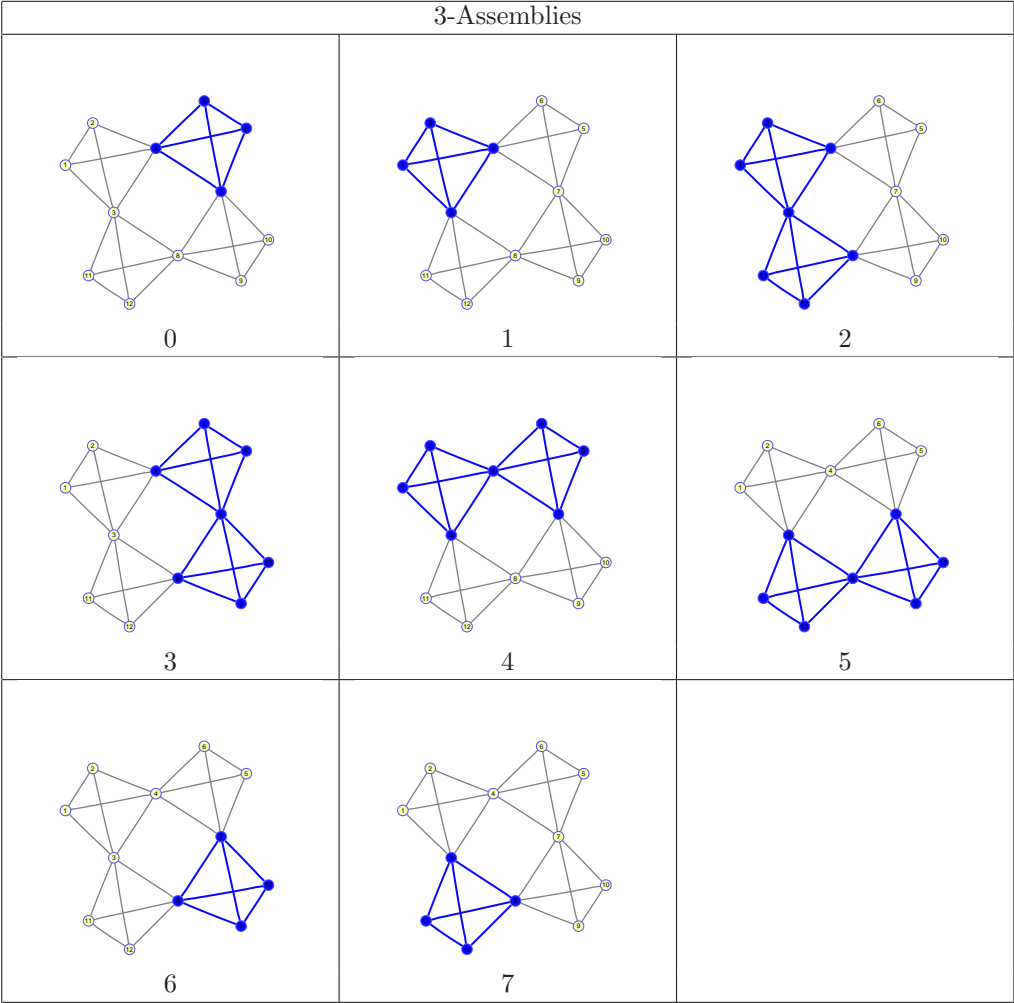
B.2.12 adj13



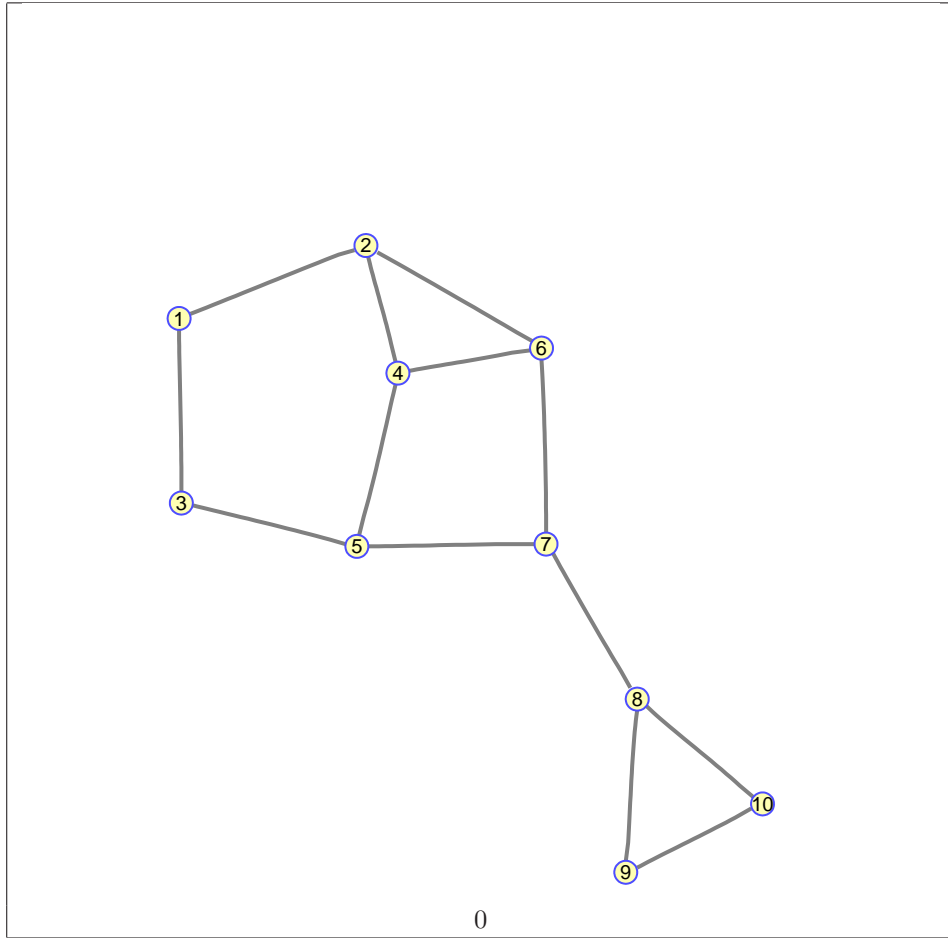
0	1	1	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	1	0	0	1	1
1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	0	1	1	0	0	0	0	0
0	0	0	1	1	0	1	0	0	0	0	0
0	0	0	1	1	1	0	1	1	1	0	0
0	0	1	0	0	0	1	0	1	1	1	1
0	0	0	0	0	0	1	1	0	1	0	0
0	0	0	0	0	0	1	1	1	0	0	0
0	0	1	0	0	0	0	1	0	0	0	1
0	0	1	0	0	0	0	1	0	0	1	0

(12)





B.2.13 adj14

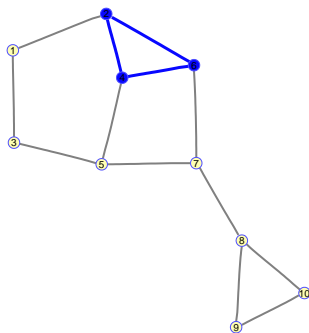


0

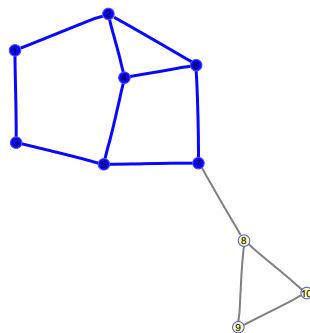
0	1	1	0	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0	0
1	0	0	0	1	0	0	0	0	0
0	1	0	0	1	1	0	0	0	0
0	0	1	1	0	0	1	0	0	0
0	1	0	1	0	0	1	0	0	0
0	0	0	0	1	1	0	1	0	0
0	0	0	0	0	0	1	0	1	1
0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	1	1	0

(13)

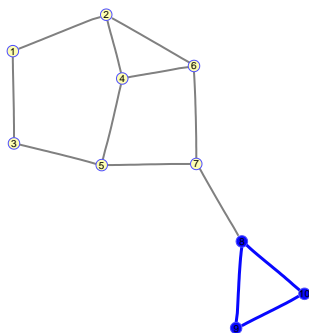
2-Assemblies



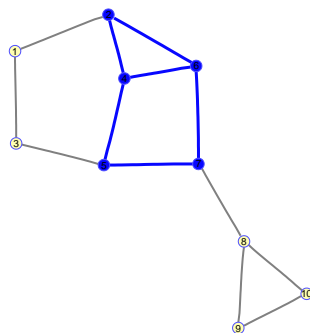
0



1

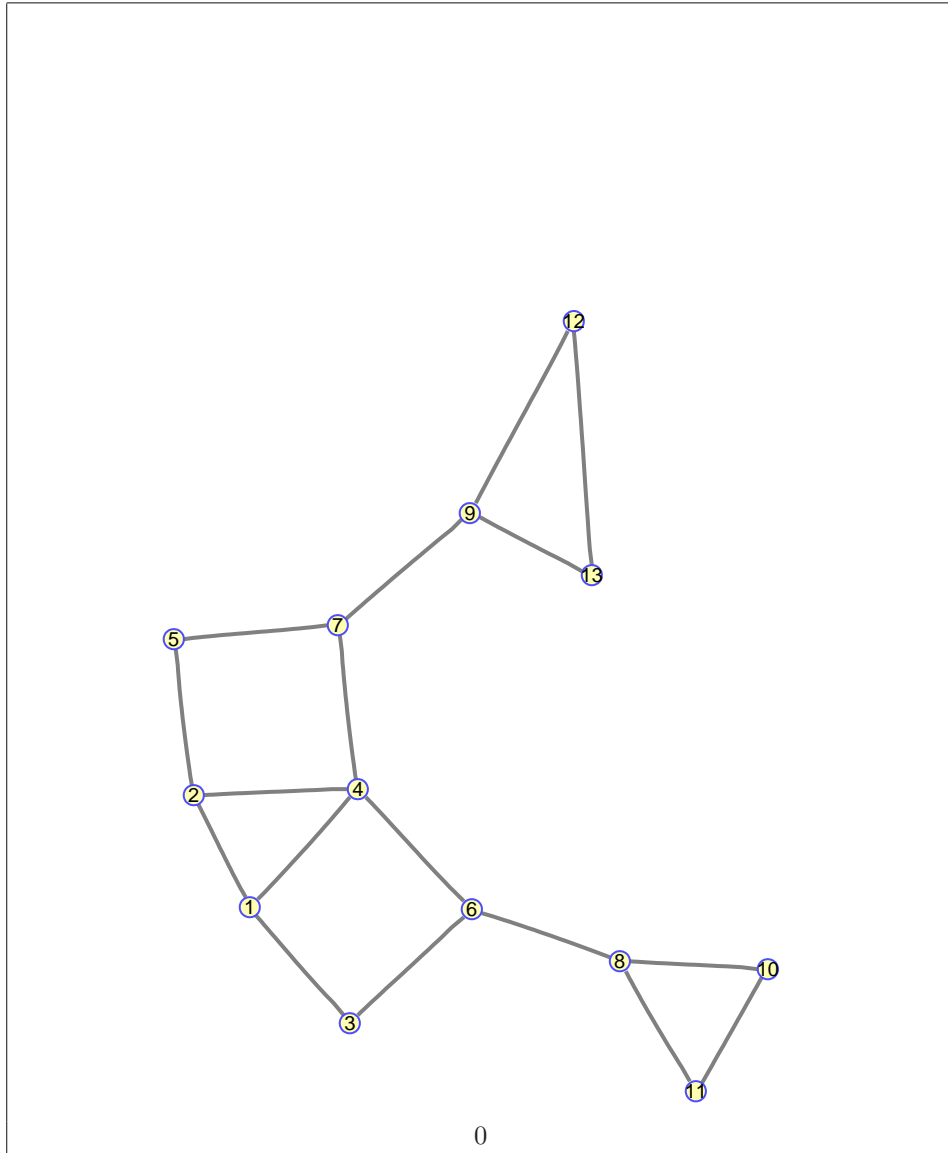


2



3

B.2.14 adj15

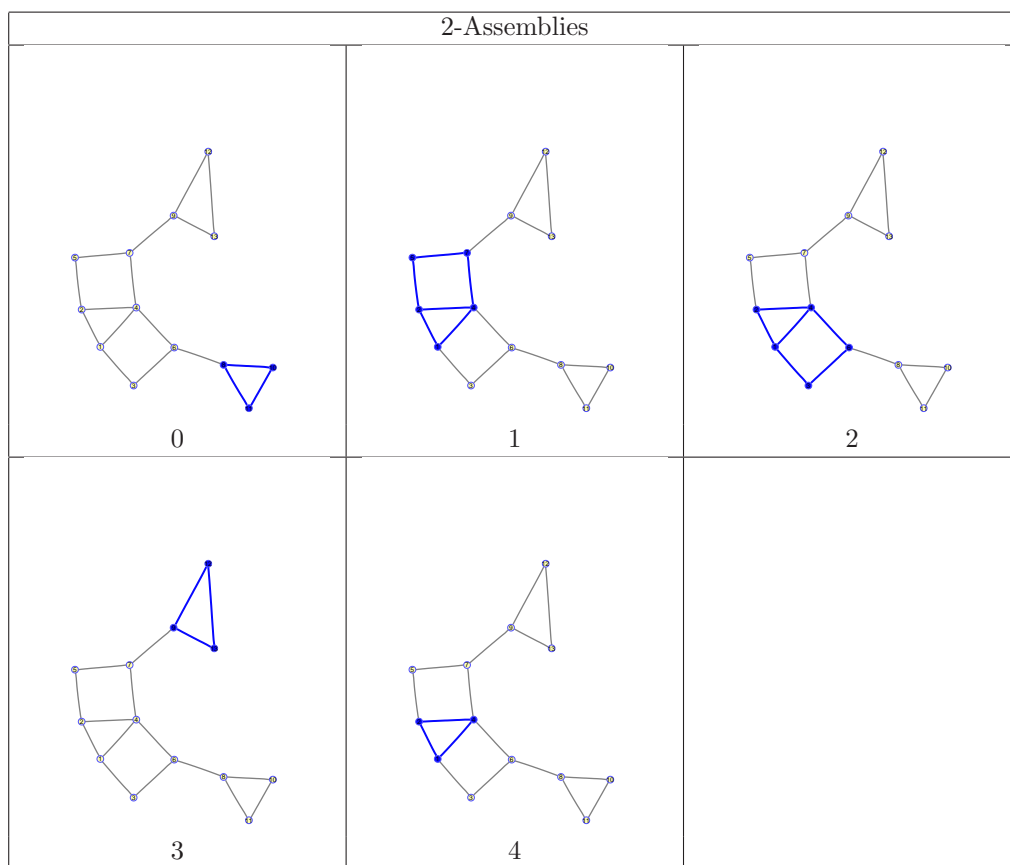


```

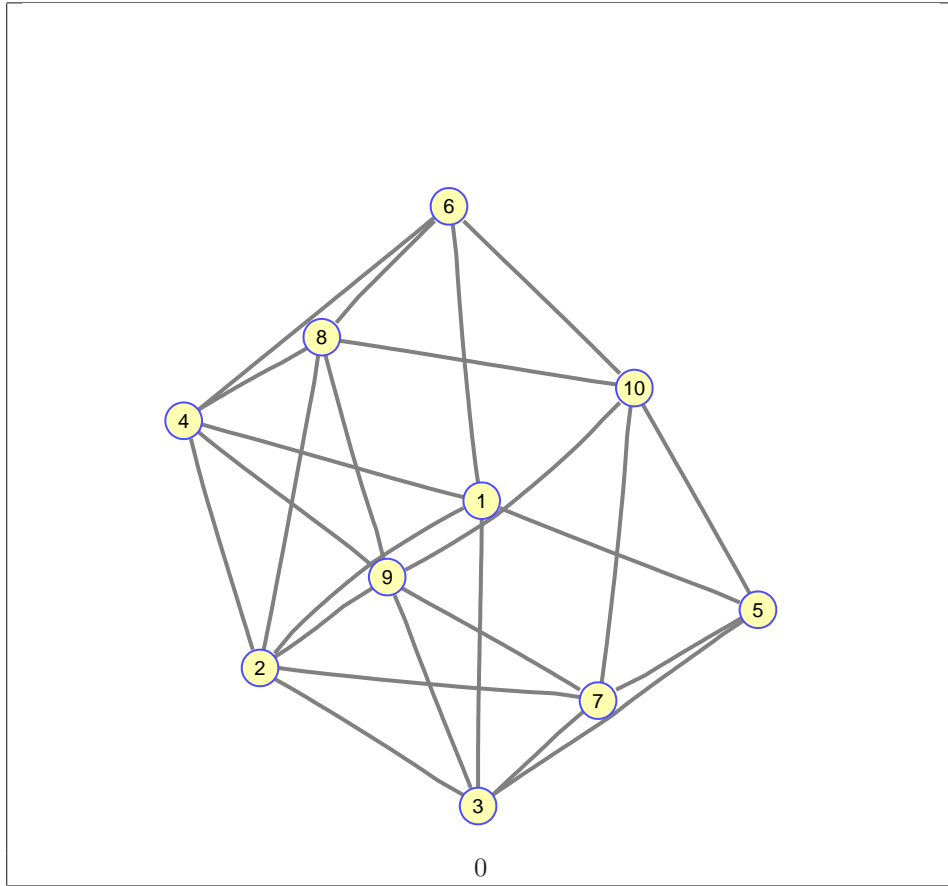
0 1 1 1 0 0 0 0 0 0 0 0
1 0 0 1 1 0 0 0 0 0 0 0
1 0 0 0 0 1 0 0 0 0 0 0
1 1 0 0 0 1 1 0 0 0 0 0
0 1 0 0 0 0 1 0 0 0 0 0
0 0 1 1 0 0 0 1 0 0 0 0
0 0 0 1 1 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0 0 1 1 0
0 0 0 0 0 0 1 0 0 0 1 1
0 0 0 0 0 0 0 1 0 0 1 0
0 0 0 0 0 0 0 1 0 1 0 0
0 0 0 0 0 0 0 0 1 0 0 1
0 0 0 0 0 0 0 0 1 0 0 1
0 0 0 0 0 0 0 0 0 1 0 0

```

(14)



B.2.15 adj16



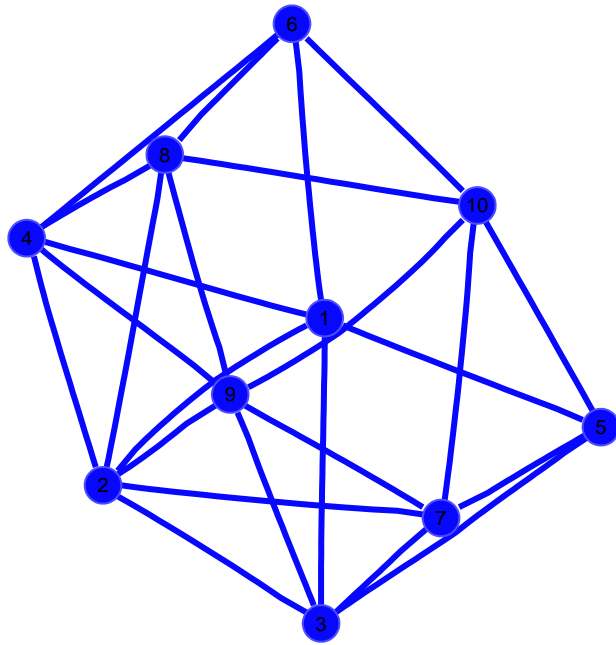
```

0 1 1 1 1 1 0 0 0 0
1 0 1 1 0 0 1 1 1 0
1 1 0 0 1 0 1 0 1 0
1 1 0 0 0 1 0 1 1 0
1 0 1 0 0 0 1 0 0 1
1 0 0 1 0 0 0 1 0 1
0 1 1 0 1 0 0 0 1 1
0 1 0 1 0 1 0 0 1 1
0 1 1 1 0 0 1 1 0 1
0 0 0 0 1 1 1 1 1 0

```

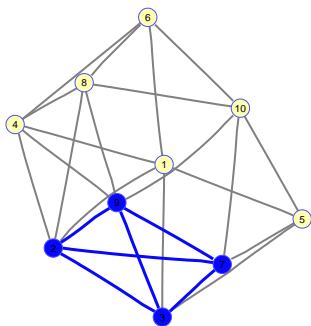
(15)

2-Assemblies

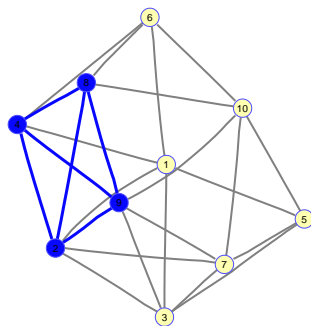


0

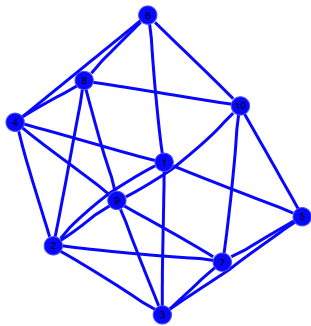
3-Assemblies



0

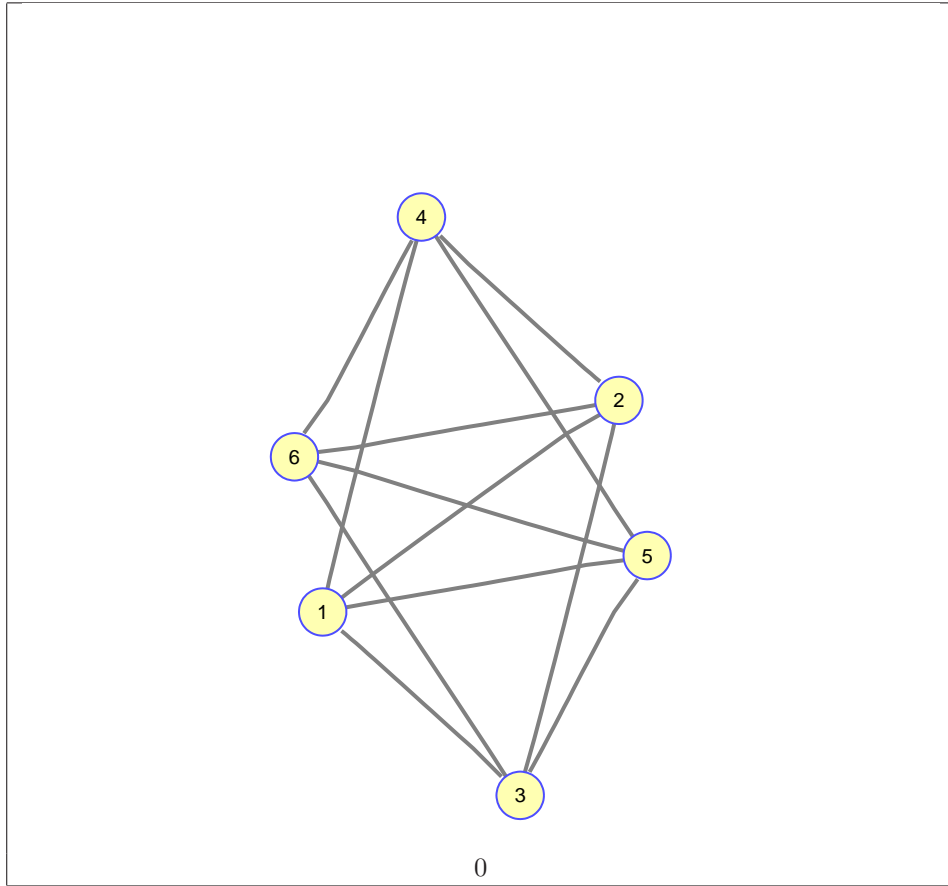


1



2

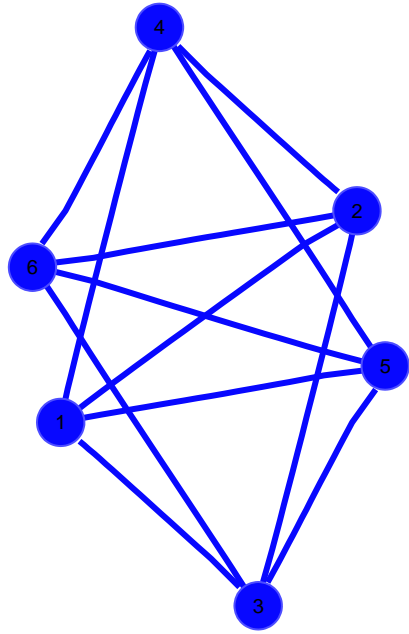
B.2.16 adj17



```
0 1 1 1 1 0
1 0 1 1 0 1
1 1 0 0 1 1
1 1 0 0 1 1
1 0 1 1 0 1
0 1 1 1 1 0
```

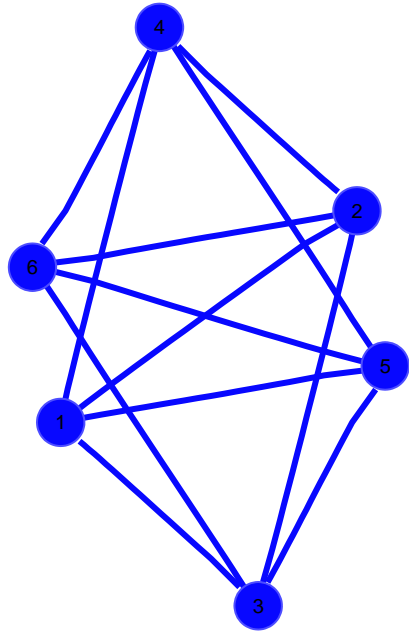
(16)

2-Assemblies



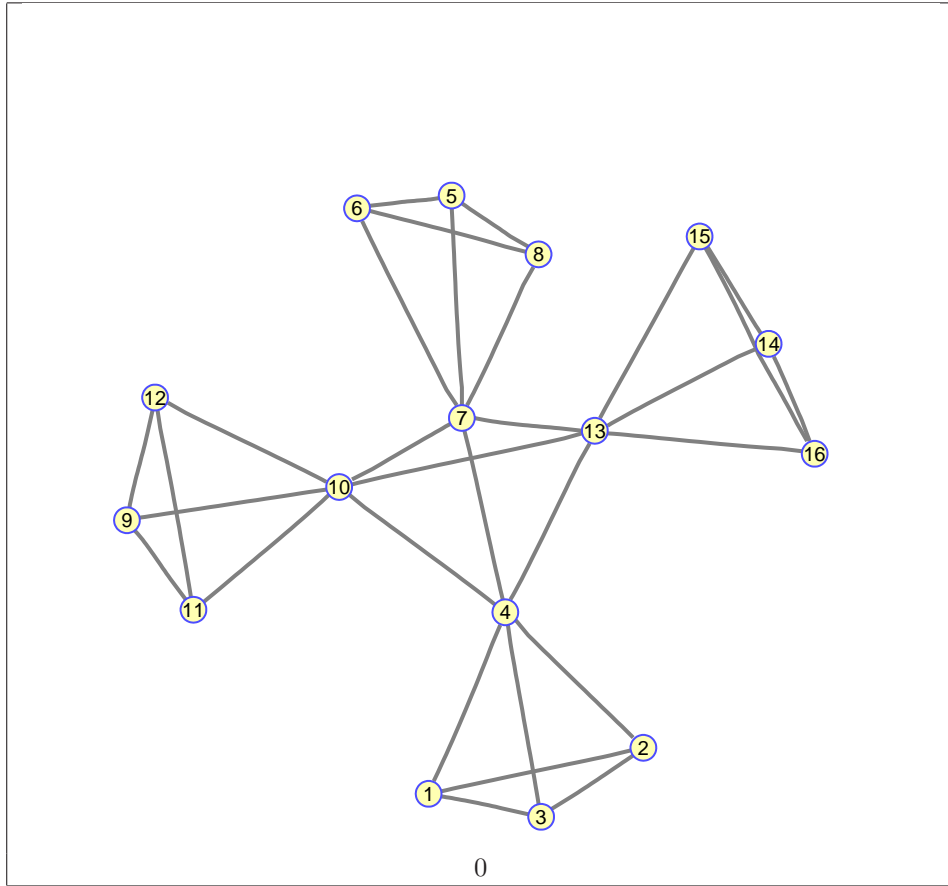
0

3-Assemblies



0

B.2.17 adj18



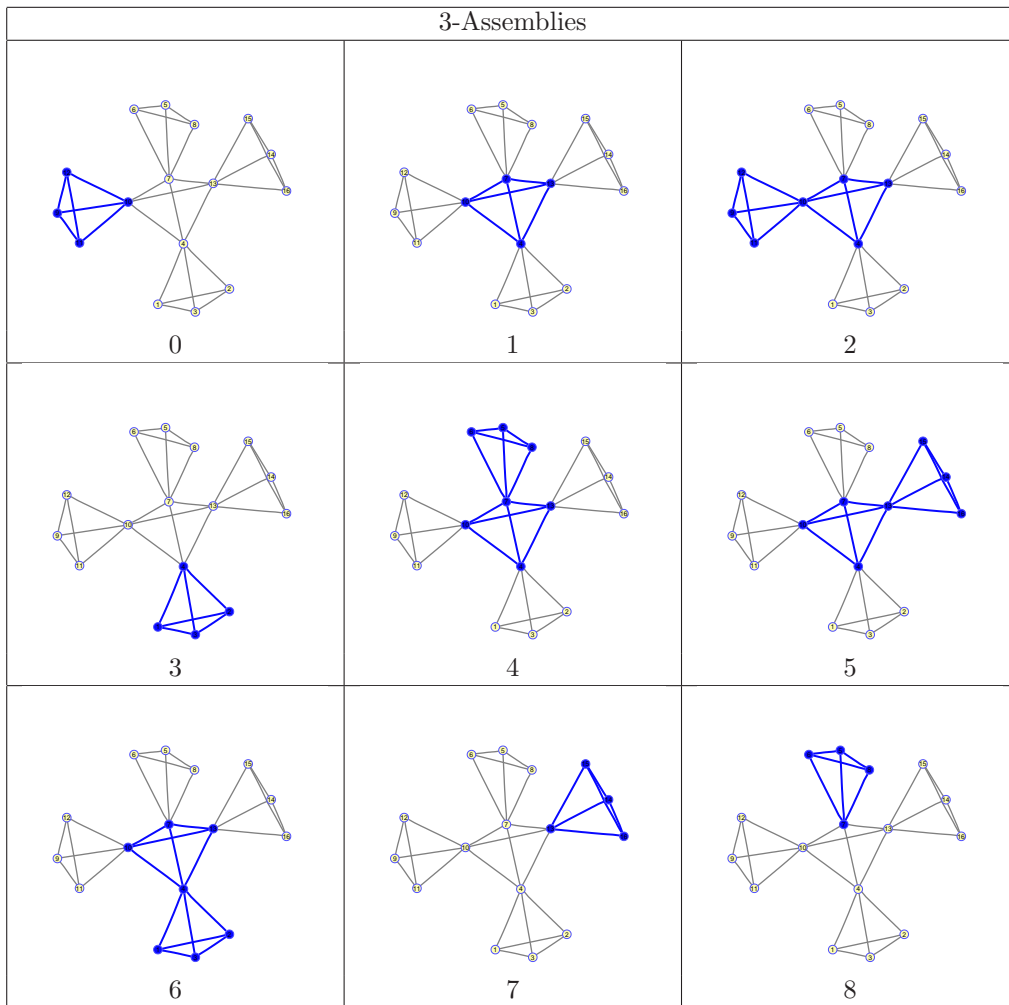
```

0 1 1 1 0 0 0 0 0 0 0 0 0 0 0
1 0 1 1 0 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 0 0 0 0 0 0 0 0
1 1 1 0 0 0 1 0 0 1 0 0 1 0 0
0 0 0 0 0 1 1 1 0 0 0 0 0 0 0
0 0 0 0 1 0 1 1 0 0 0 0 0 0 0
0 0 0 1 1 1 0 1 0 1 0 0 1 0 0
0 0 0 0 1 1 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 1 1 0 0 0
0 0 0 1 0 0 1 0 1 0 1 1 1 0 0
0 0 0 0 0 0 0 0 0 1 1 0 1 0 0
0 0 0 0 0 0 0 0 0 1 1 1 0 0 0
0 0 0 1 0 0 1 0 0 1 0 0 0 1 1
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 1 1 1

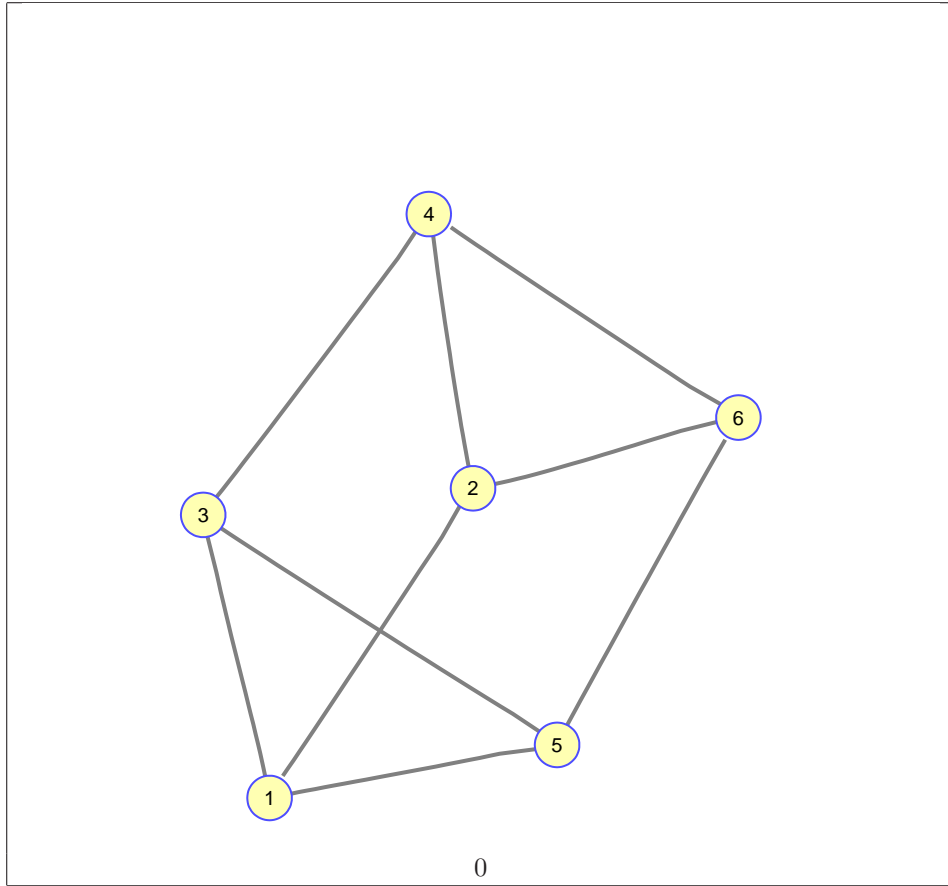
```

(17)

3-Assemblies



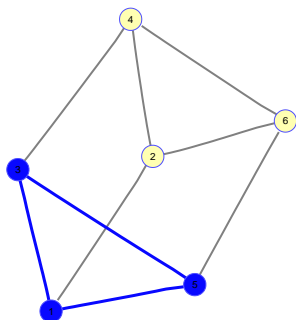
B.2.18 circle3



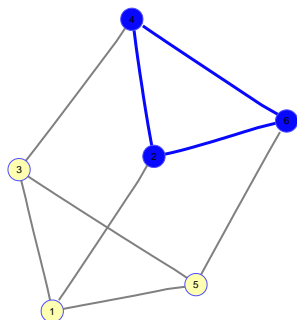
```
0 1 1 0 1 0
1 0 0 1 0 1
1 0 0 1 1 0
0 1 1 0 0 1
1 0 1 0 0 1
0 1 0 1 1 0
```

(18)

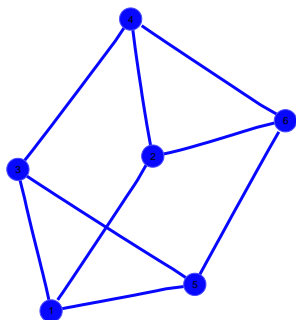
2-Assemblies



0

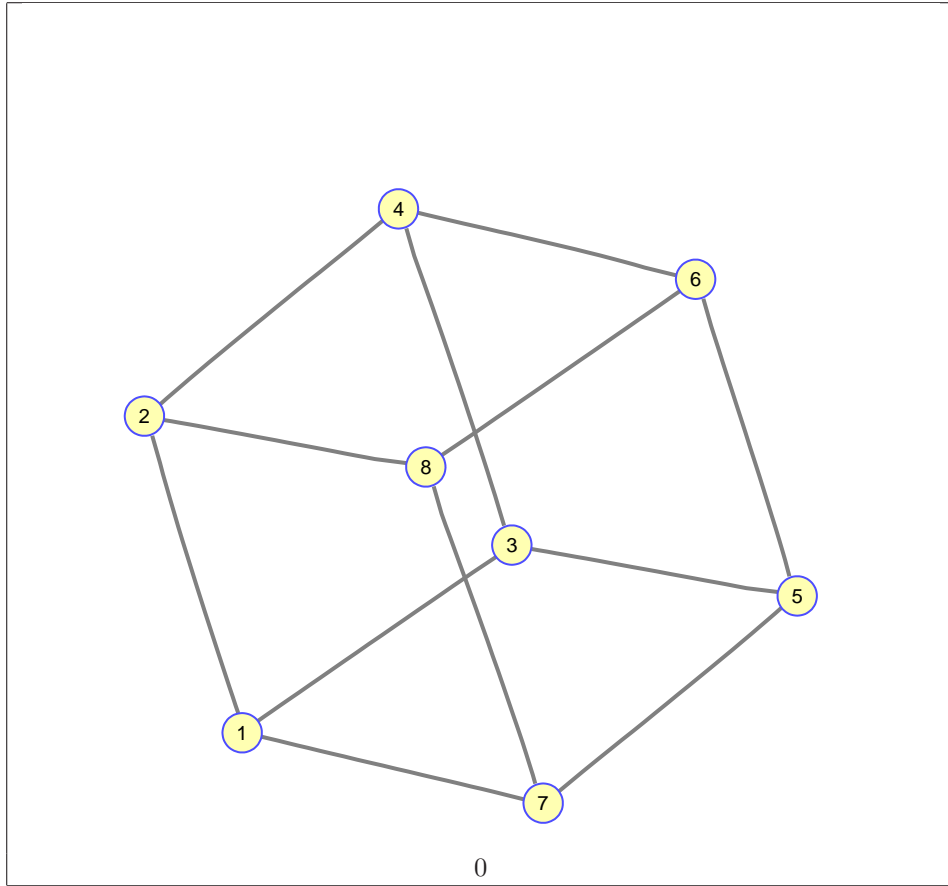


1



2

B.2.19 circle4

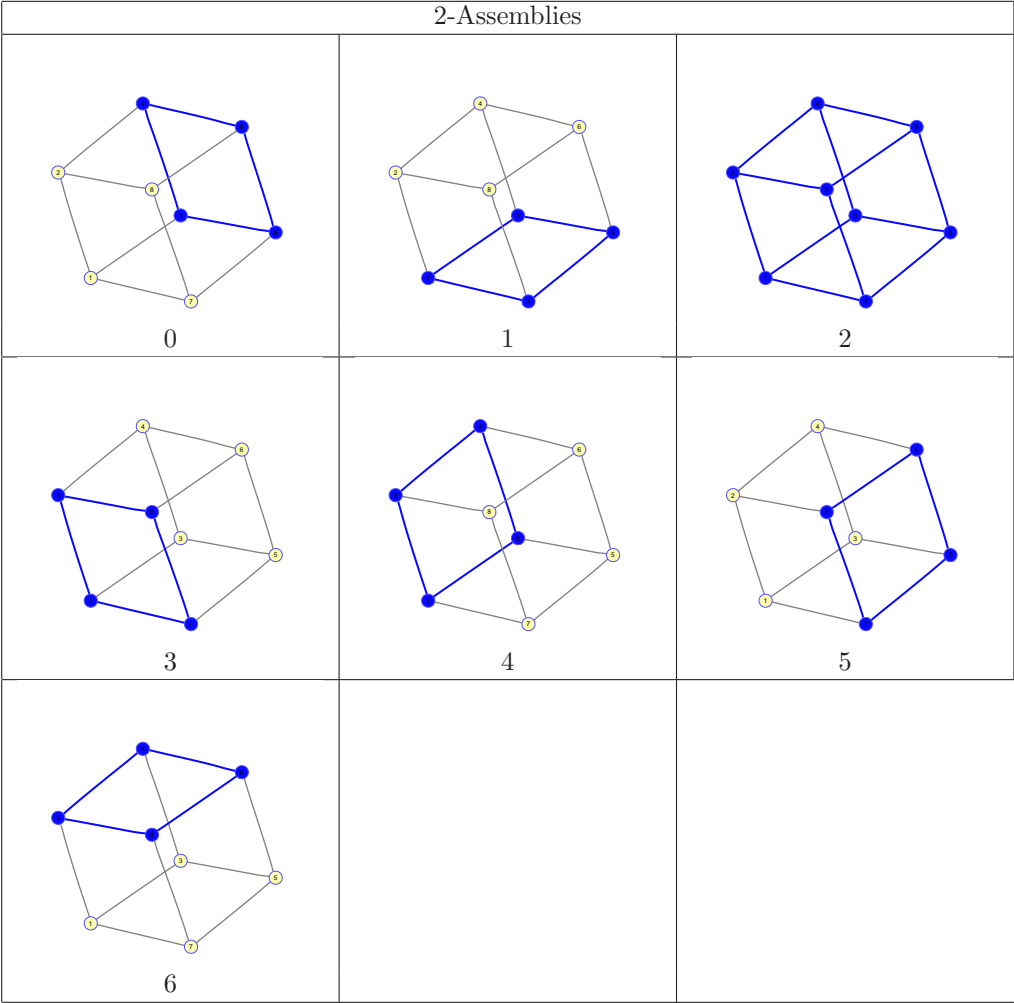


```

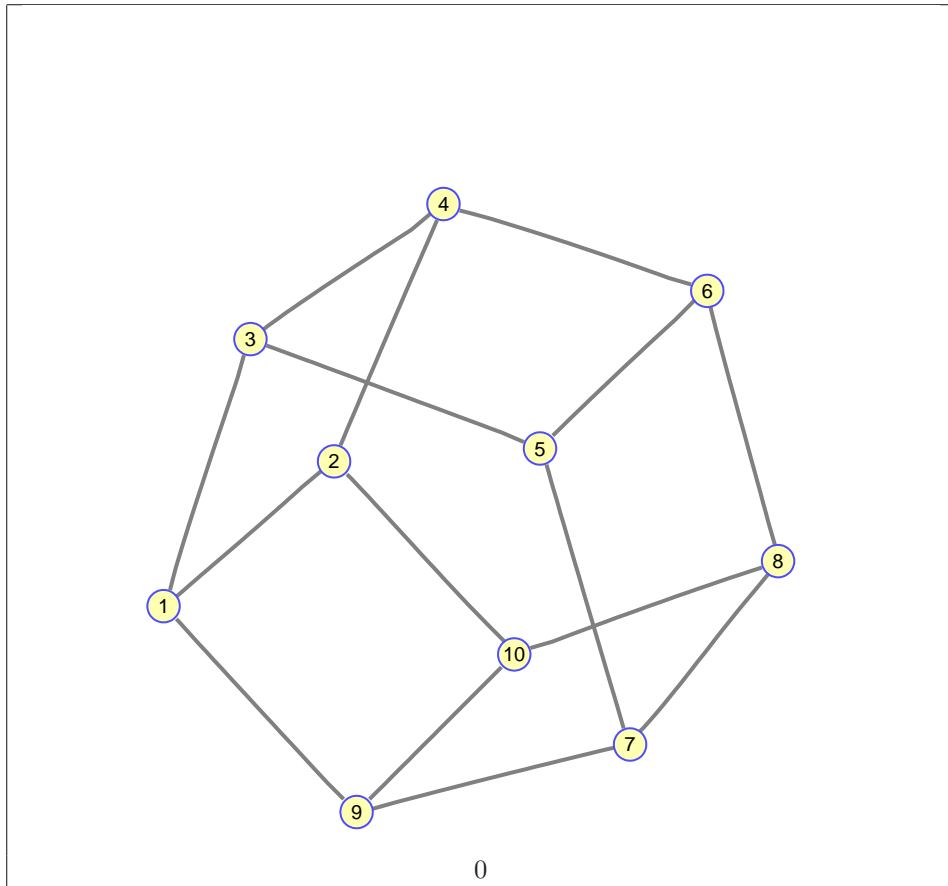
0 1 1 0 0 0 1 0
1 0 0 1 0 0 0 1
1 0 0 1 1 0 0 0
0 1 1 0 0 1 0 0
0 0 1 0 0 1 1 0
0 0 0 1 1 0 0 1
1 0 0 0 1 0 0 1
0 1 0 0 0 1 1 0

```

(19)

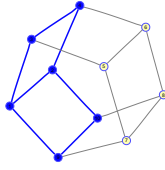
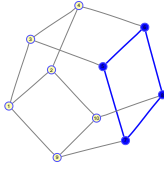
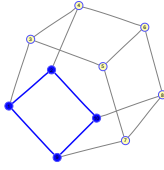
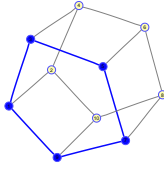
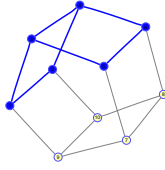
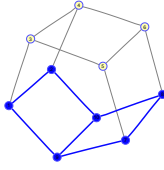
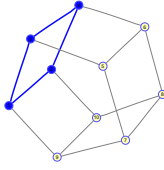
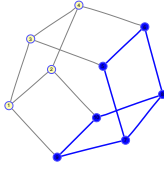
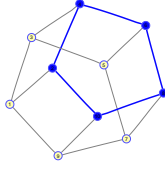
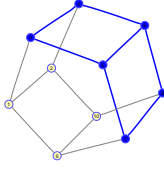
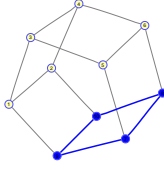
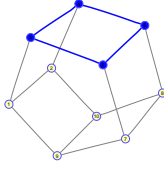
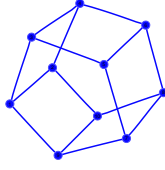


B.2.20 circle5

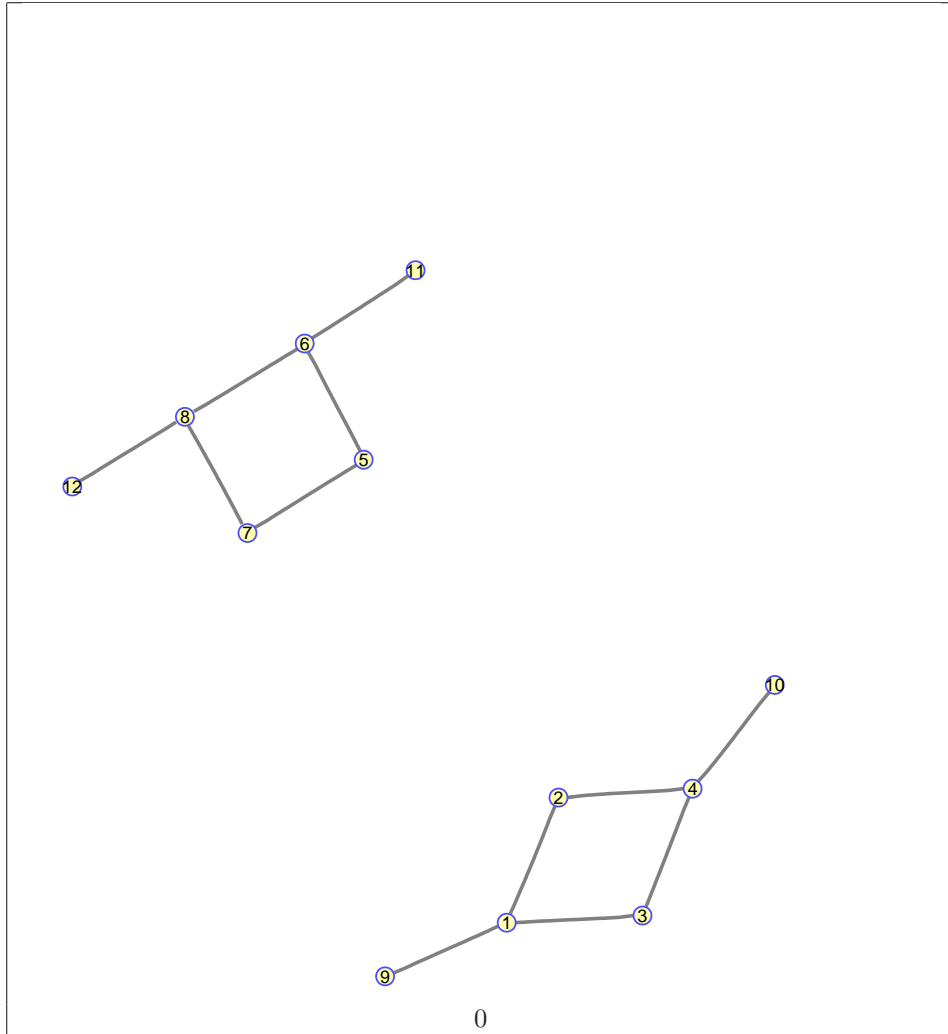


0	1	1	0	0	0	0	0	1	0
1	0	0	1	0	0	0	0	0	1
1	0	0	1	1	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0
0	0	1	0	0	1	1	0	0	0
0	0	0	1	1	0	0	1	0	0
0	0	0	0	1	0	0	1	1	0
0	0	0	0	0	1	1	0	0	1
1	0	0	0	0	0	1	0	0	1
0	1	0	0	0	0	0	1	1	0

(20)

2-Assemblies			
 0	 1	 2	 3
 4	 5	 6	 7
 8	 9	 10	 11
 12			

B.2.21 disconnected

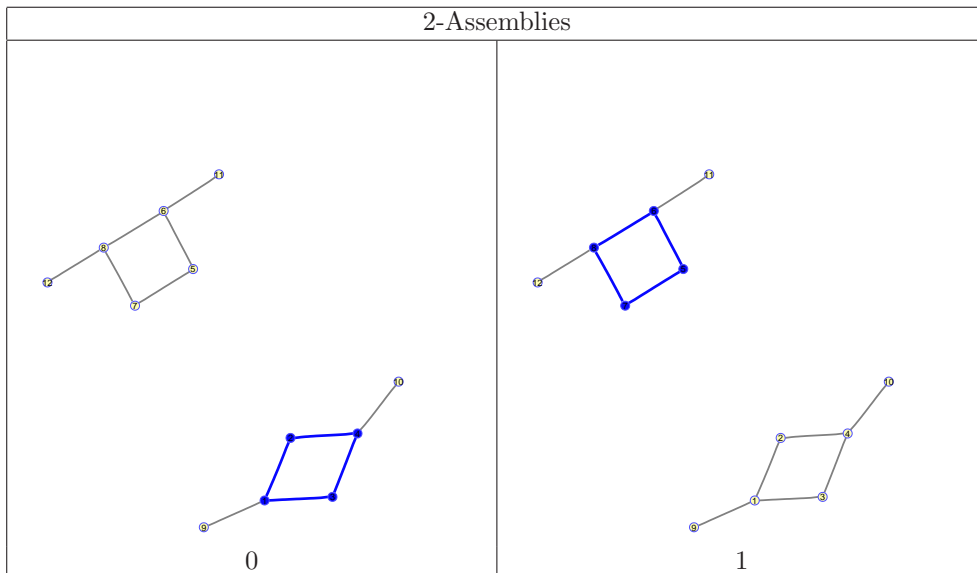


```

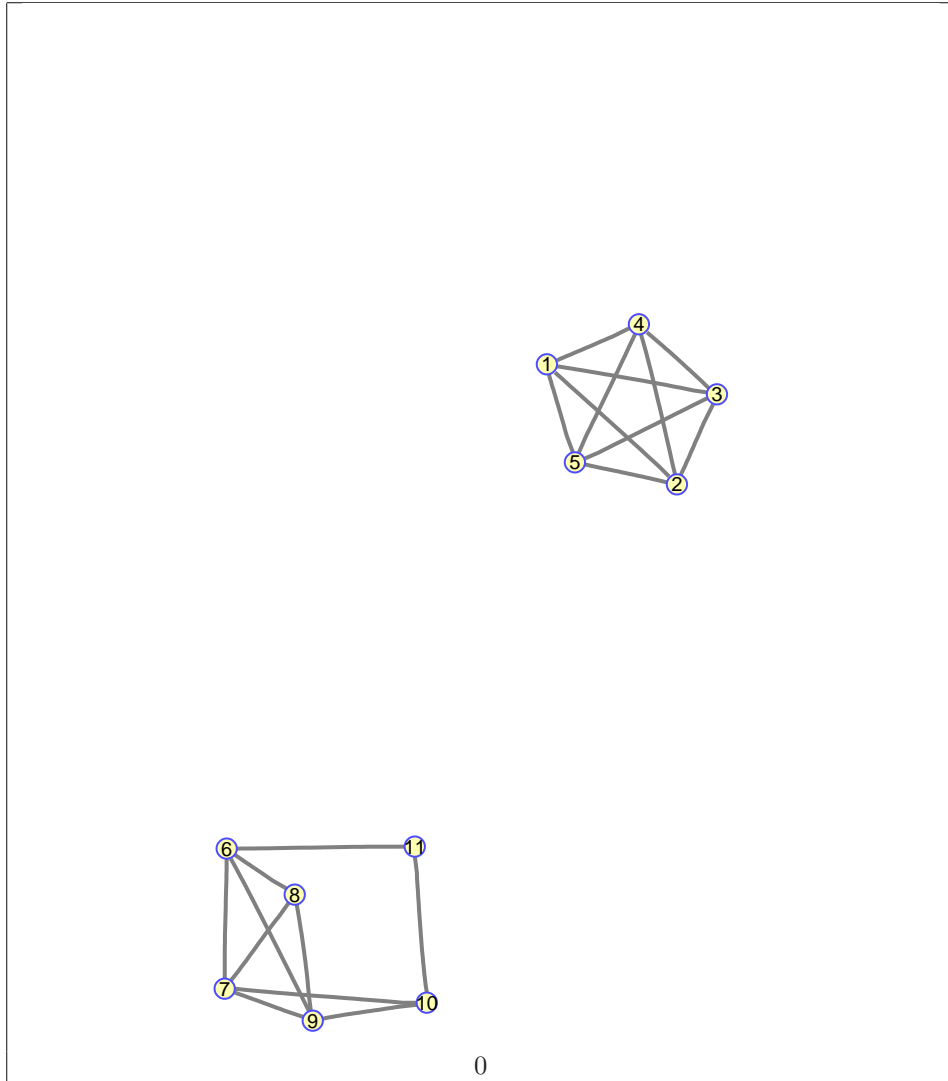
0 1 1 0 0 0 0 0 1 0 0 0
1 0 0 1 0 0 0 0 0 0 0 0
1 0 0 1 0 0 0 0 0 0 0 0
0 1 1 0 0 0 0 0 0 1 0 0
0 0 0 0 0 1 1 0 0 0 0 0
0 0 0 0 1 0 0 1 0 0 1 0
0 0 0 0 1 0 0 1 0 0 0 0
0 0 0 0 0 1 1 0 0 0 0 1
1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0

```

(21)



B.2.22 disconnected2

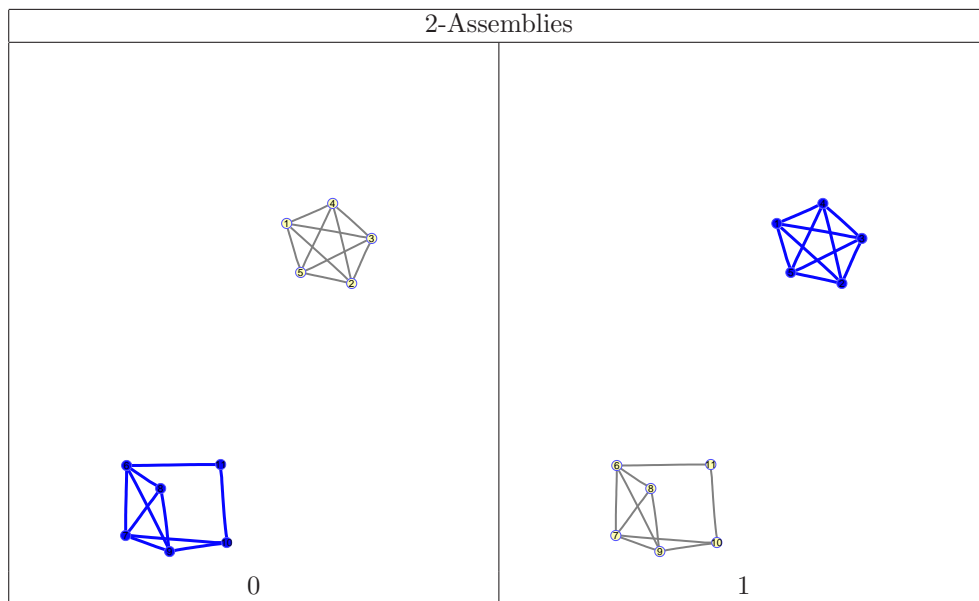


```

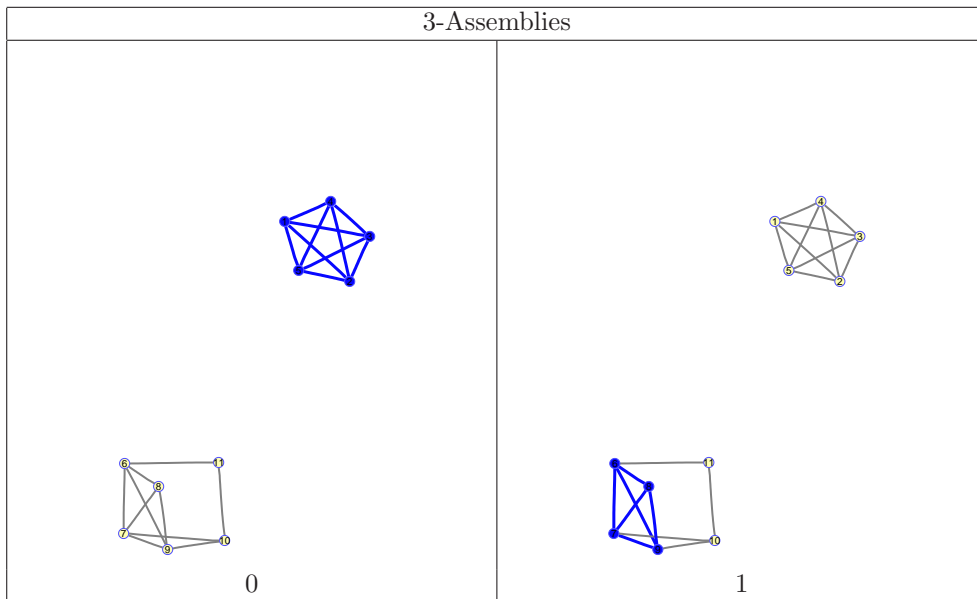
0 1 1 1 1 0 0 0 0 0 0
1 0 1 1 1 0 0 0 0 0 0
1 1 0 1 1 0 0 0 0 0 0
1 1 1 0 1 0 0 0 0 0 0
1 1 1 1 0 0 0 0 0 0 0
0 0 0 0 0 0 1 1 1 0 1
0 0 0 0 0 1 0 1 1 1 0
0 0 0 0 0 1 1 0 1 0 0
0 0 0 0 0 1 1 1 0 1 0
0 0 0 0 0 0 1 0 1 0 1
0 0 0 0 0 1 0 0 0 1 0

```

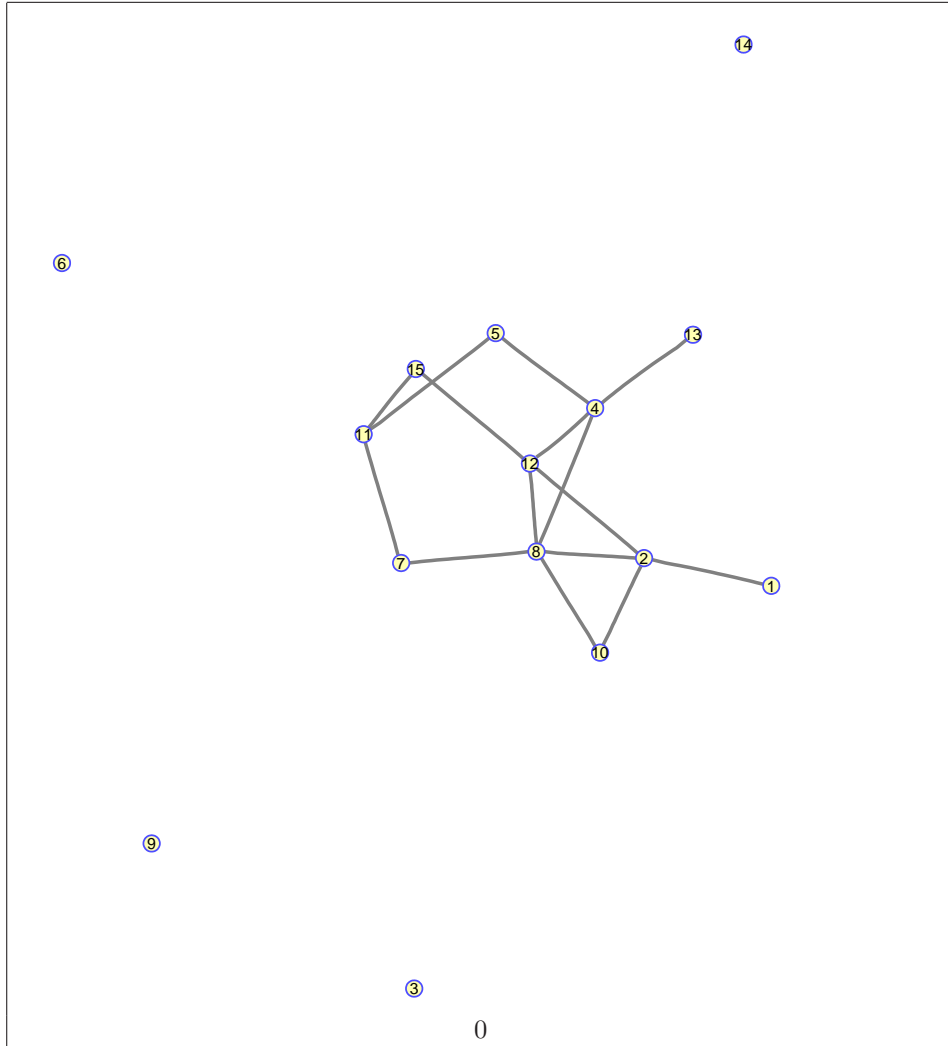
(22)



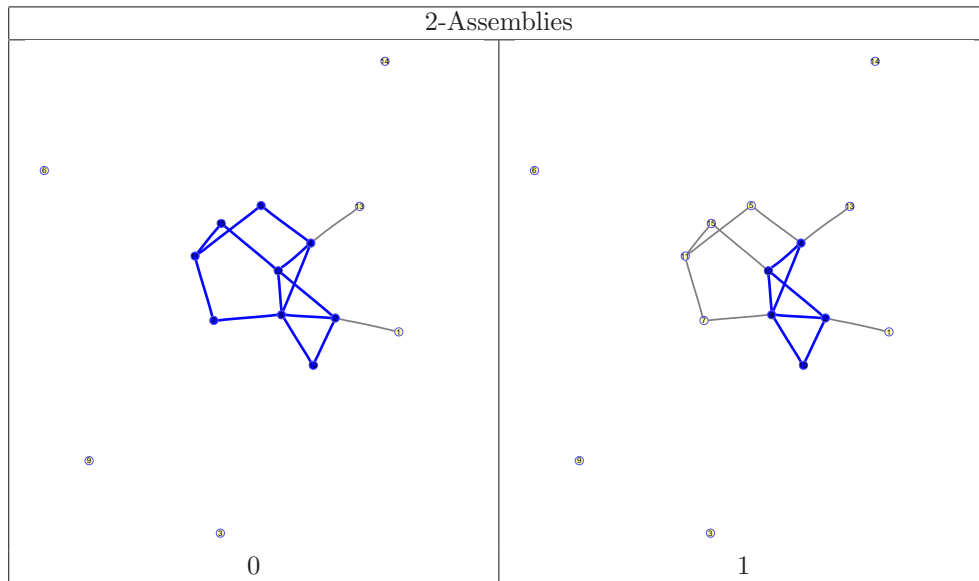
3-Assemblies



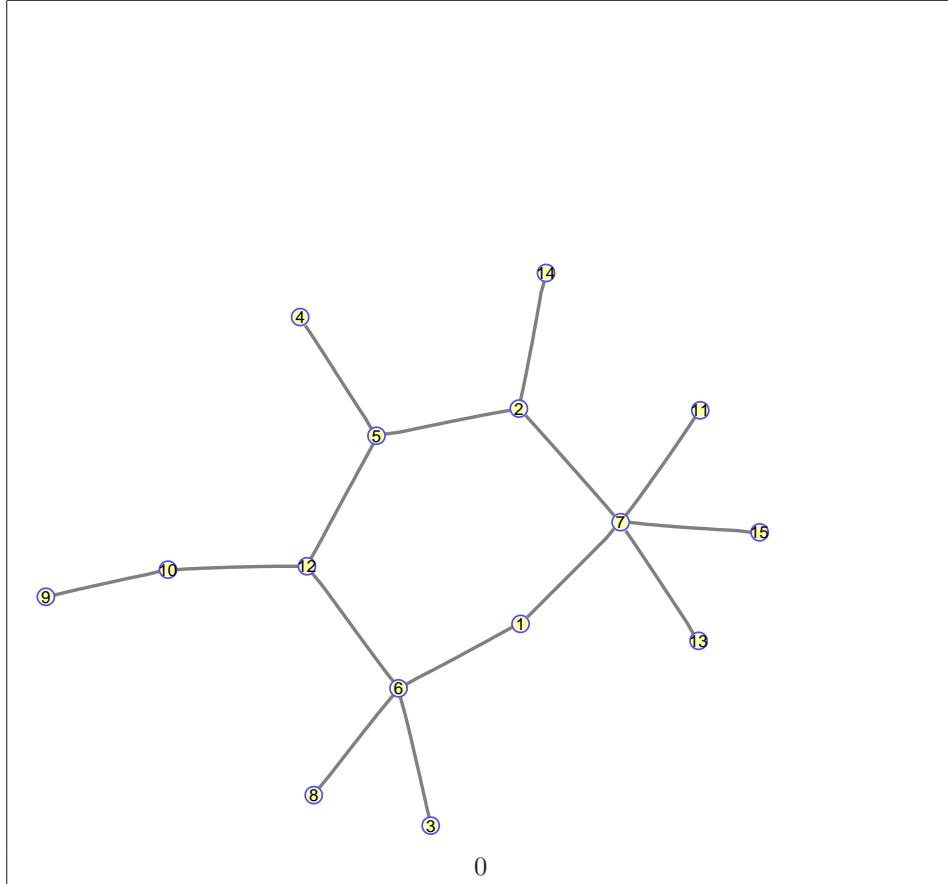
B.2.23 `randomGraph(15,0.075,undirected)-1`



$$\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{23}$$



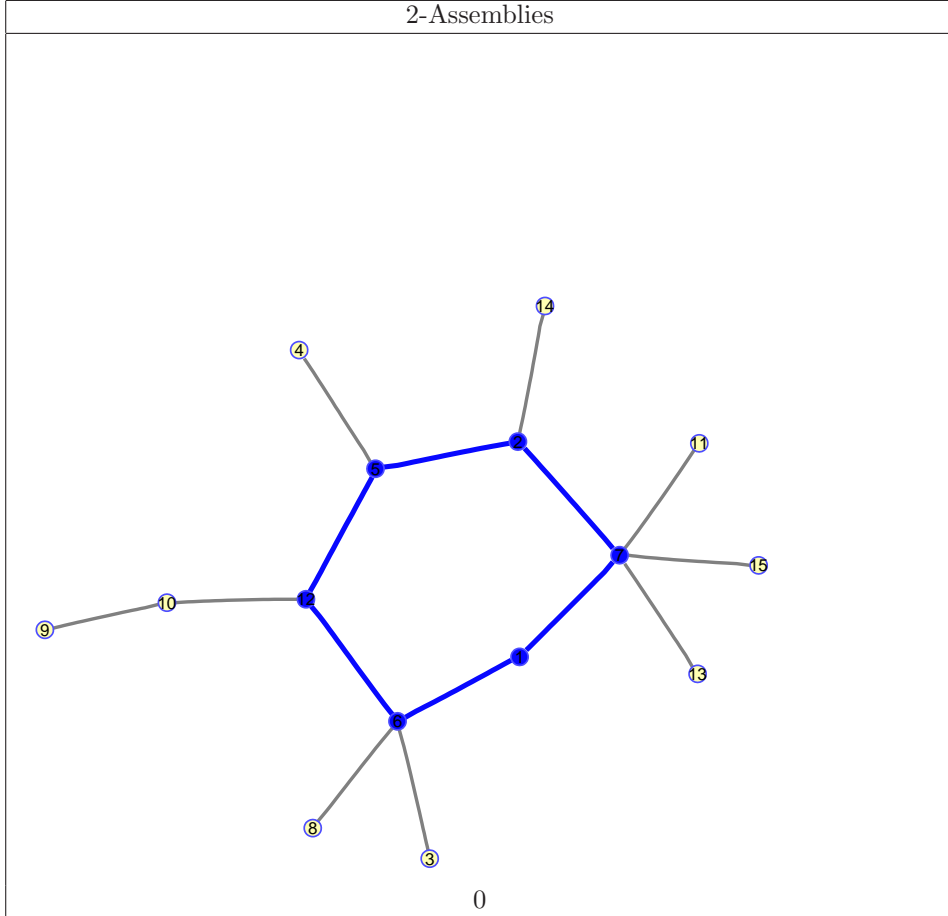
B.2.24 randomGraph(15,0.075,undirected)-2



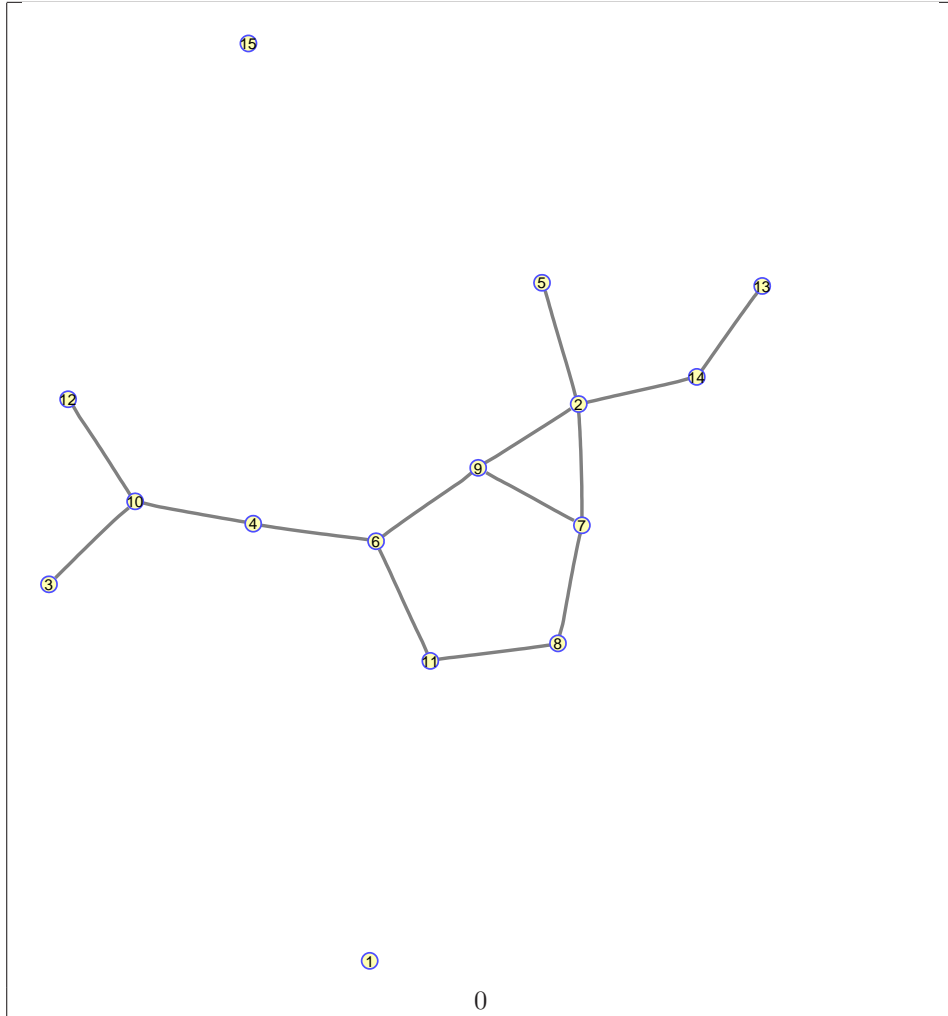
0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	1	0	0	0	1	0	0	0
1	1	0	0	0	0	0	0	0	0	1	0	1	0	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

(24)

2-Assemblies



B.2.25 `randomGraph(15,0.075,undirected)-3`

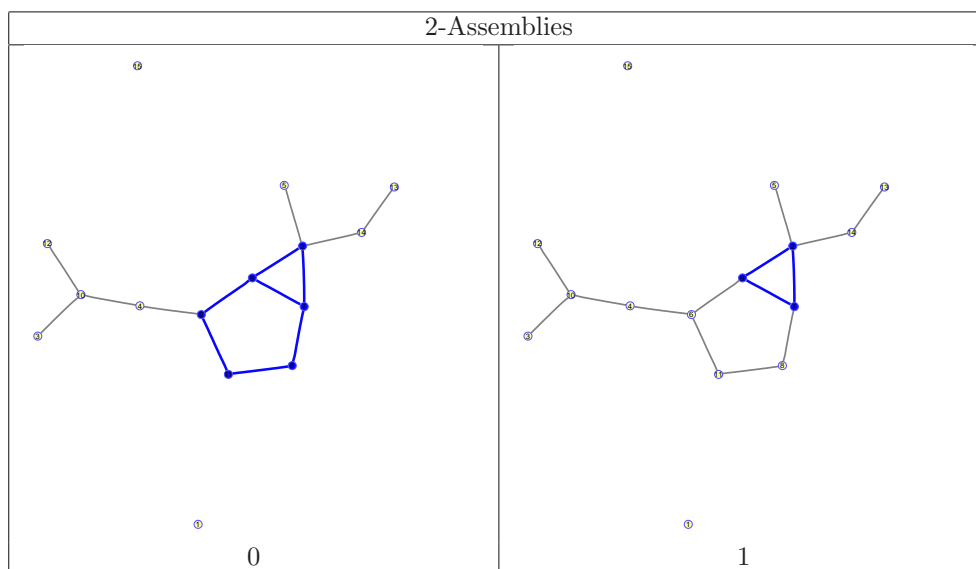


```

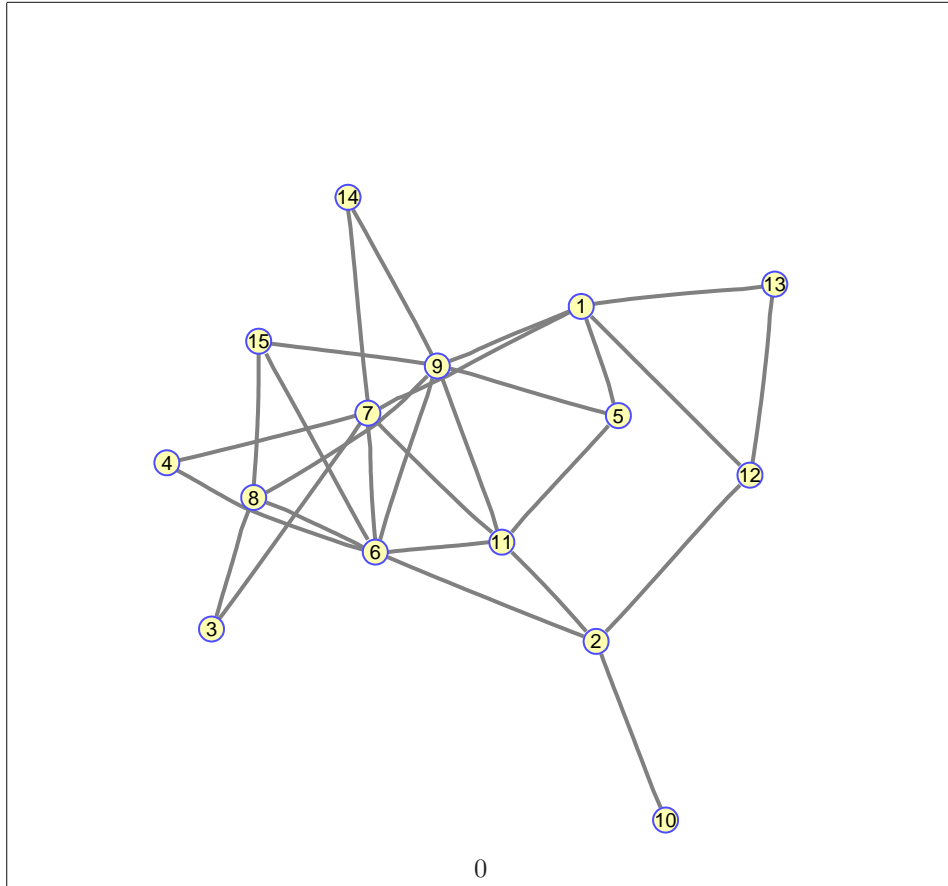
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 1 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 1 0 0 0 1 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 1 0 1 0 0 0 0
0 1 0 0 0 0 0 1 1 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 1 0 0 0 0
0 1 0 0 0 1 1 0 0 0 0 0 0 0 0
0 0 1 1 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 1 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

(25)



B.2.26 randomGraph(15,0.15,undirected)-1

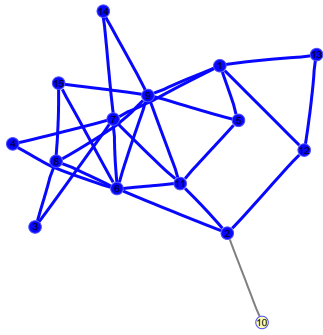


0

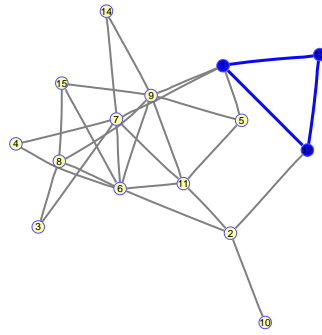
0	0	0	0	1	0	1	0	1	0	0	1	1	0	0
0	0	0	0	0	1	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1	0	1	0	0	0	0
0	1	0	1	0	0	1	1	1	0	1	0	0	0	1
1	0	1	1	0	1	0	0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0	1	0	0	0	0	0	1
1	0	0	0	1	1	0	1	0	0	1	0	0	1	1
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	1	1	0	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	1	1	0	0	0	0	0	0

(26)

2-Assemblies

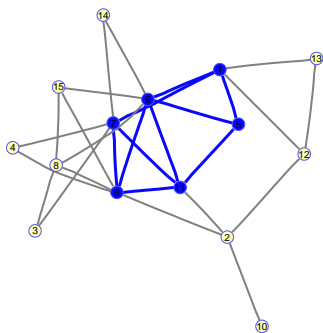


0

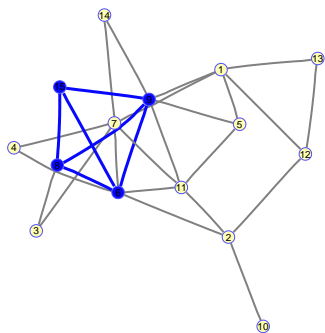


1

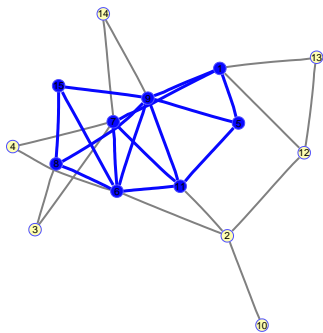
3-Assemblies



0

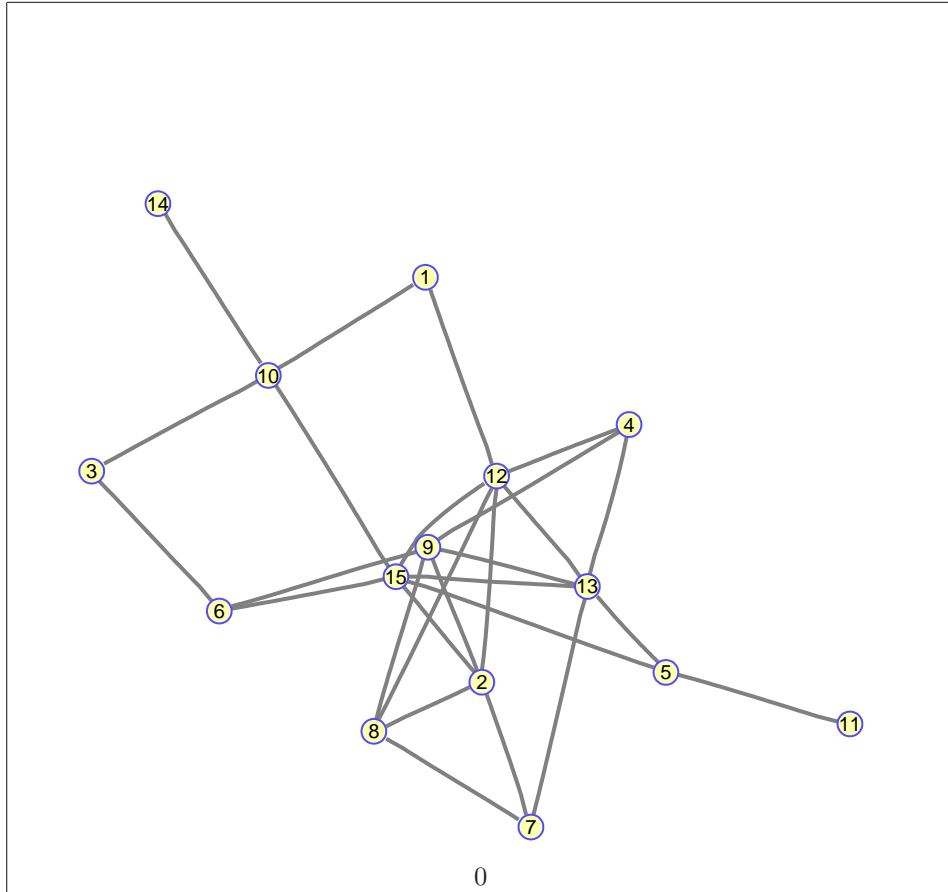


1



2

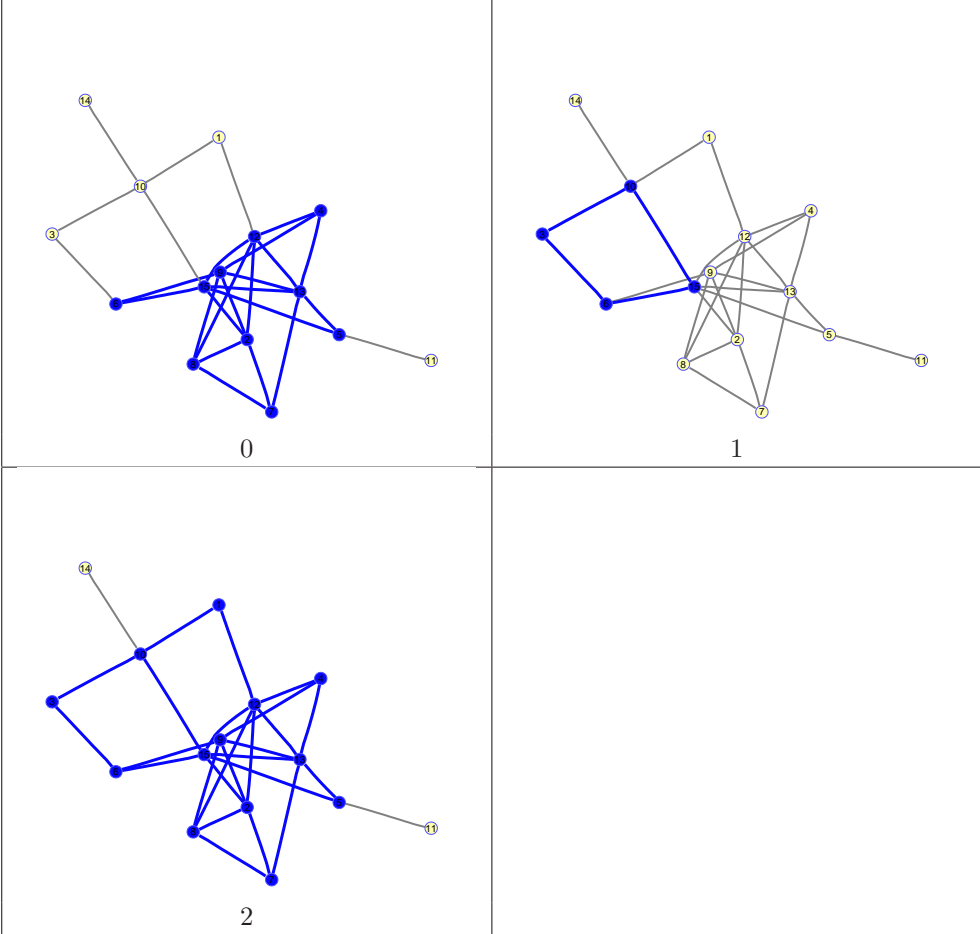
B.2.27 randomGraph(15,0.15,undirected)-2



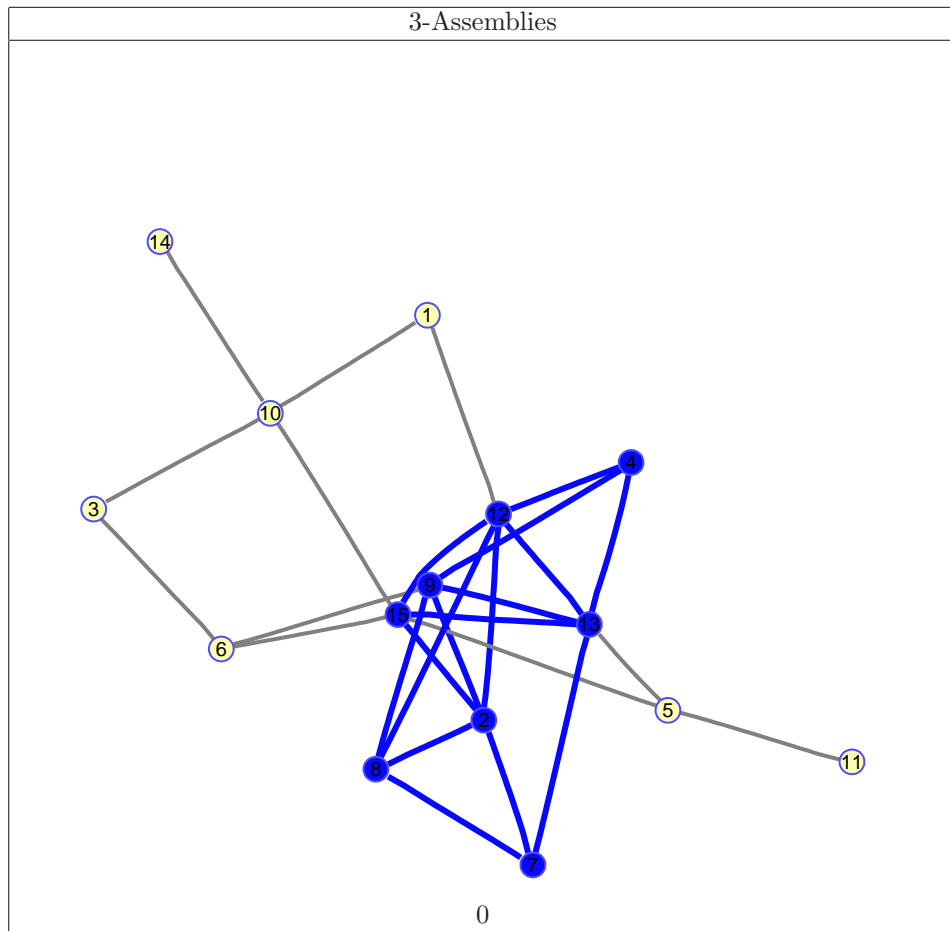
0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	1
0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0
0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1
0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0
0	1	0	0	0	0	1	0	1	0	0	1	0	0	0	0
0	1	0	1	0	1	0	1	0	0	0	0	0	1	0	0
1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	1	0	0	0	0	0	1	0	1
0	0	0	1	1	0	1	0	1	0	0	1	0	0	0	1
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	1	0	0	1	1	0	0	0	1	0	1	1	0	0	0

(27)

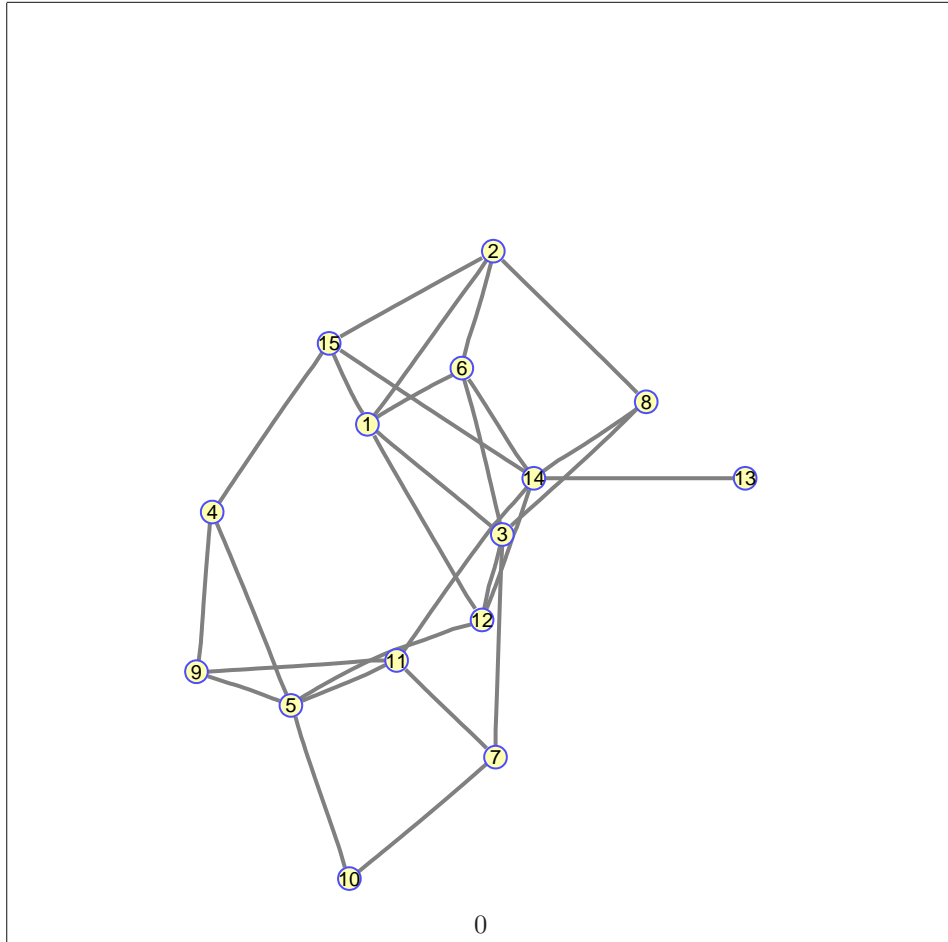
2-Assemblies



3-Assemblies



B.2.28 `randomGraph(15,0.15,undirected)-3`



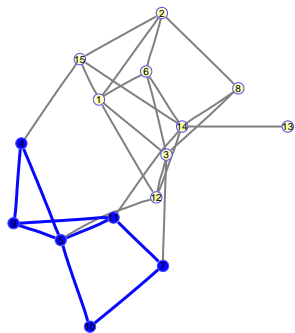
```

0 1 1 0 0 1 0 0 0 0 0 1 0 0 1
1 0 0 0 0 1 0 1 0 0 0 0 0 0 1
1 0 0 0 0 1 1 1 0 0 0 1 0 0 0
0 0 0 0 1 0 0 0 1 0 0 0 0 0 1
0 0 0 1 0 0 0 0 1 1 1 1 0 0 0
1 1 1 0 0 0 0 0 0 0 0 0 0 1 0
0 0 1 0 0 0 0 0 0 1 1 0 0 0 0
0 1 1 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 1 1 0 0 0 0 0 1 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 1 0 0 0 0 1 0
1 0 1 0 1 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 1 0 1 0 0 1 1 1 0 1
1 1 0 1 0 0 0 0 0 0 0 0 0 1 0

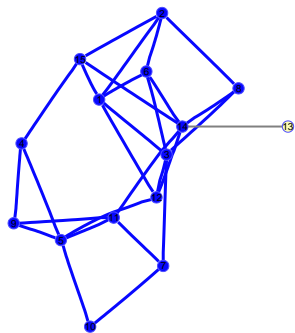
```

(28)

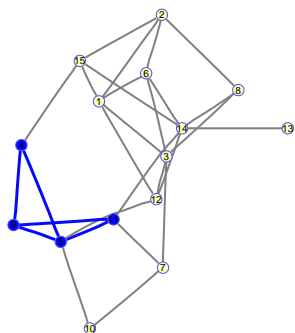
2-Assemblies



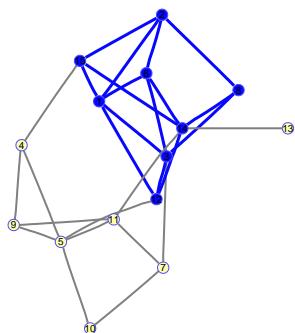
0



1

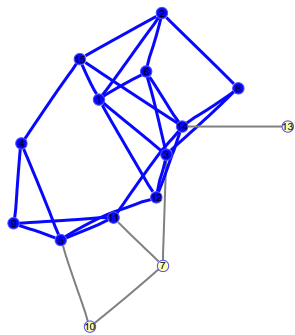


2

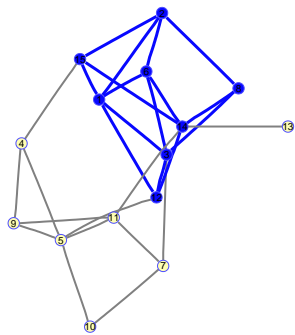


3

3-Assemblies

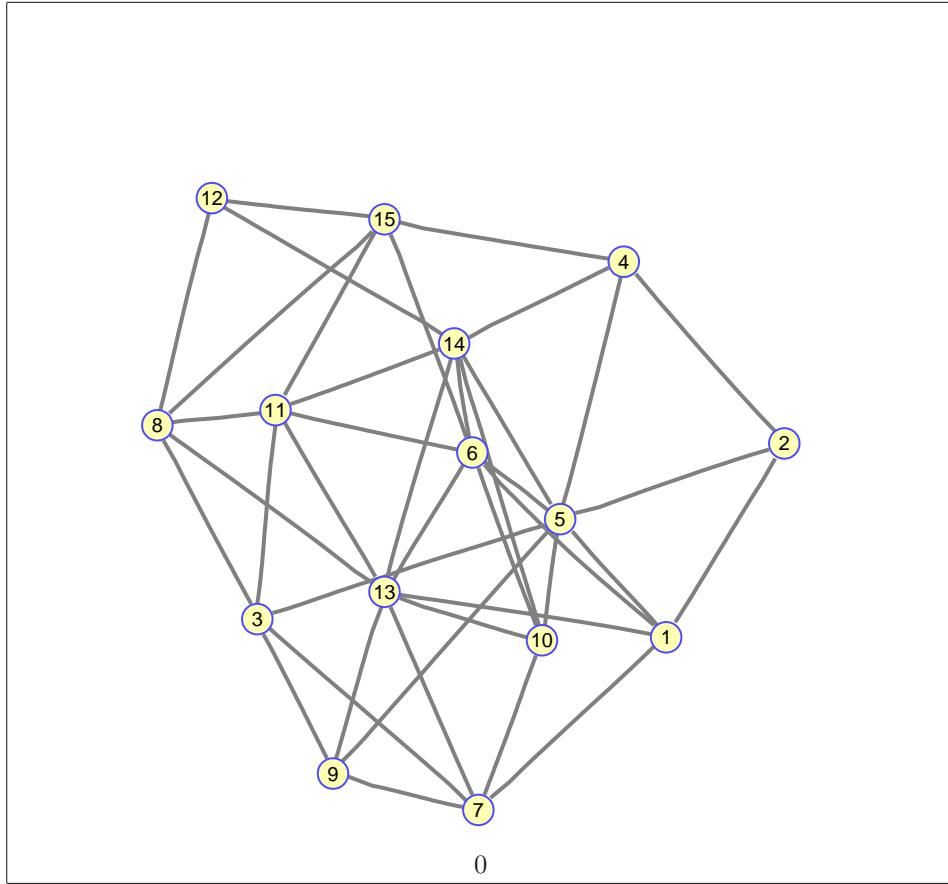


0



1

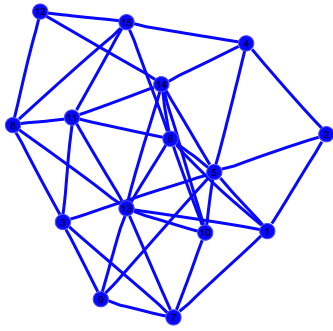
B.2.29 randomGraph(15,0.25,undirected)-1



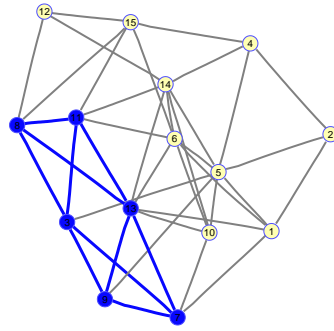
0	1	0	0	1	1	1	0	0	0	0	0	1	0	0
1	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	1	1	0	1	0	0	0	0
0	1	0	0	1	0	0	0	0	0	0	0	0	1	1
1	1	1	1	0	1	0	0	1	1	0	0	0	1	0
1	0	0	0	1	0	0	0	0	1	1	0	1	1	1
1	0	1	0	0	0	0	0	1	1	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0	1	1	1	0	1
0	0	1	0	1	0	1	0	0	0	0	0	1	0	0
0	0	0	0	1	1	1	0	0	0	0	0	1	1	0
0	0	1	0	0	1	0	1	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
1	0	0	0	0	1	1	1	1	1	1	0	0	1	0
0	0	0	1	1	1	0	0	0	1	1	1	1	0	0
0	0	0	1	0	1	0	1	0	0	1	1	0	0	0

(29)

3-Assemblies

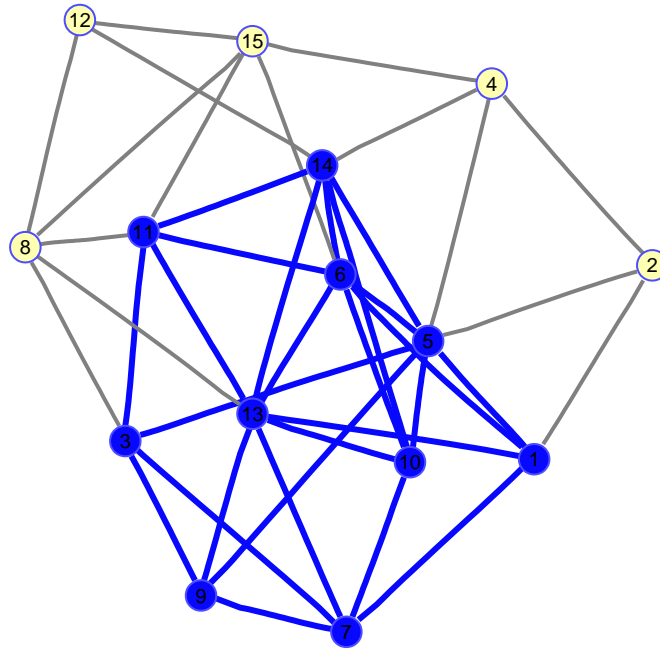


0



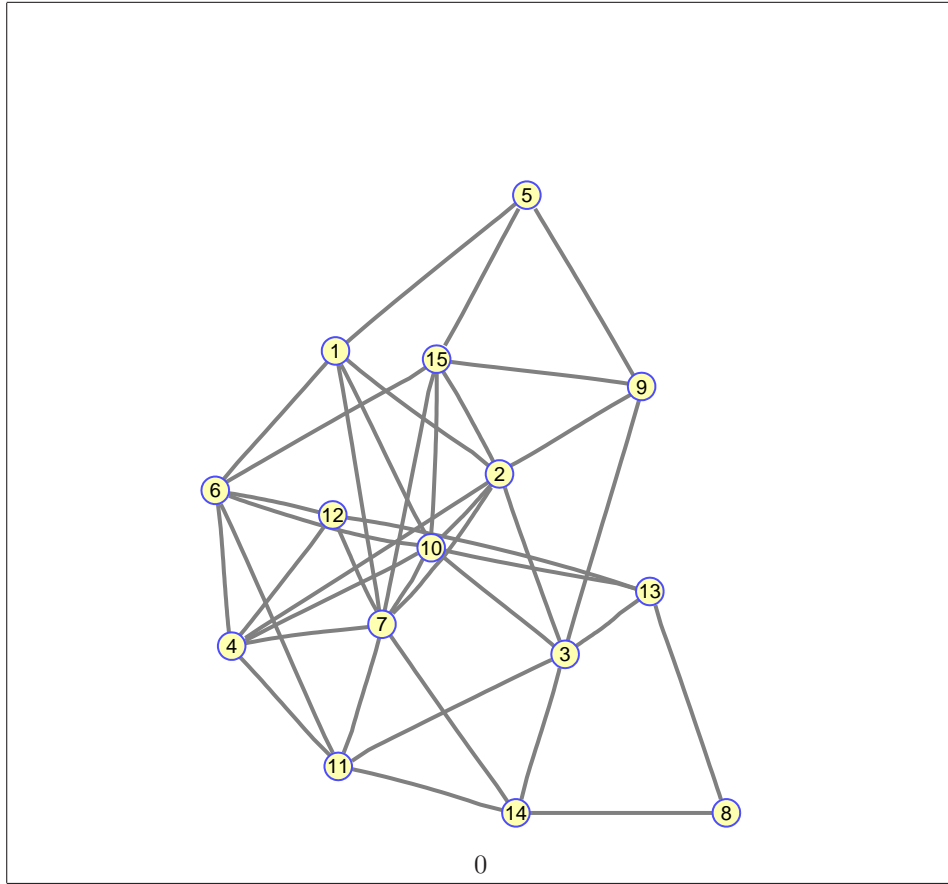
1

4-Assemblies



0

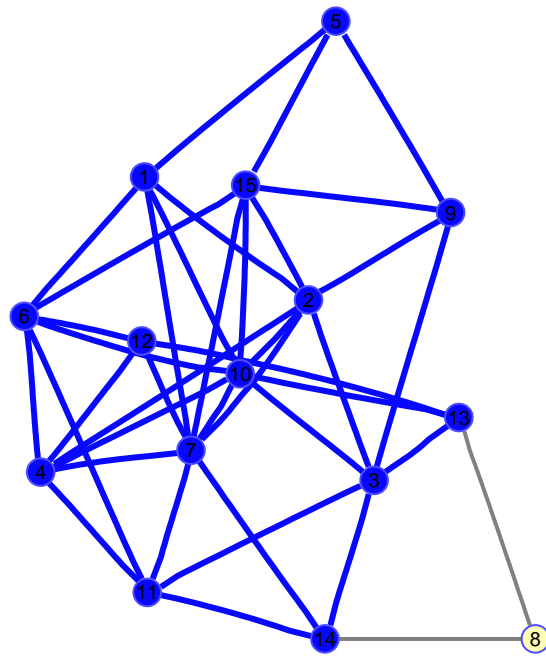
B.2.30 randomGraph(15,0.25,undirected)-2



0	1	0	0	1	1	1	0	0	1	0	0	0	0	0
1	0	1	1	0	0	1	0	1	1	0	0	0	0	1
0	1	0	0	0	0	0	0	1	1	1	0	1	1	0
0	1	0	0	0	1	1	0	0	1	1	1	0	0	0
1	0	0	0	0	0	0	0	1	0	0	0	0	0	1
1	0	0	1	0	0	0	0	0	1	1	1	0	0	1
1	1	0	1	0	0	0	0	0	1	1	1	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
0	1	1	0	1	0	0	0	0	0	0	0	0	0	1
1	1	1	1	0	1	1	0	0	0	0	0	1	0	1
0	0	1	1	0	1	1	0	0	0	0	0	0	1	0
0	0	0	1	0	1	1	0	0	0	0	0	1	0	0
0	0	1	0	0	0	0	1	0	1	0	1	0	0	0
0	0	1	0	0	0	1	1	0	0	1	0	0	0	0
0	1	0	0	1	1	1	0	1	1	0	0	0	0	0

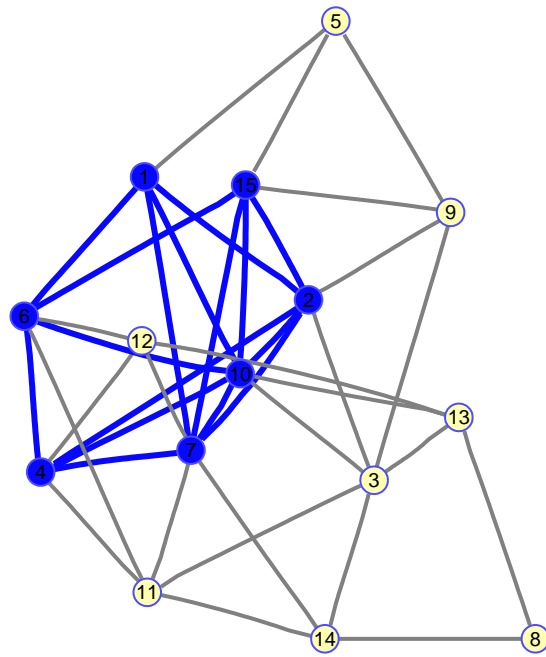
(30)

3-Assemblies



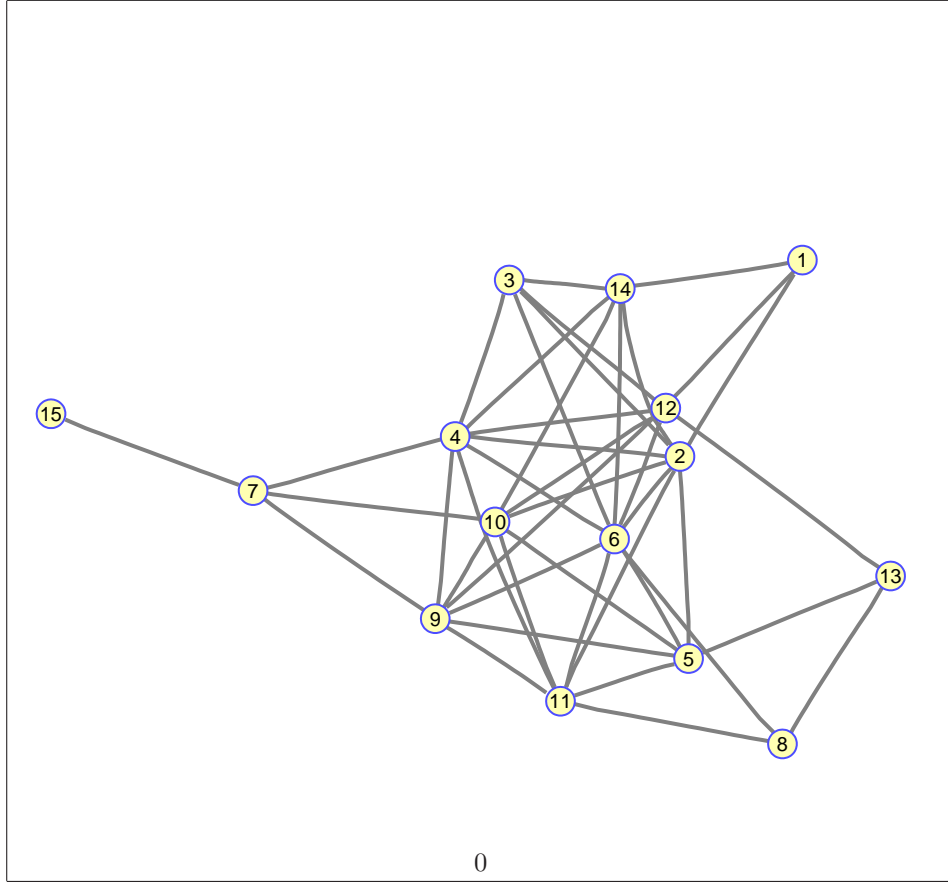
0

4-Assemblies



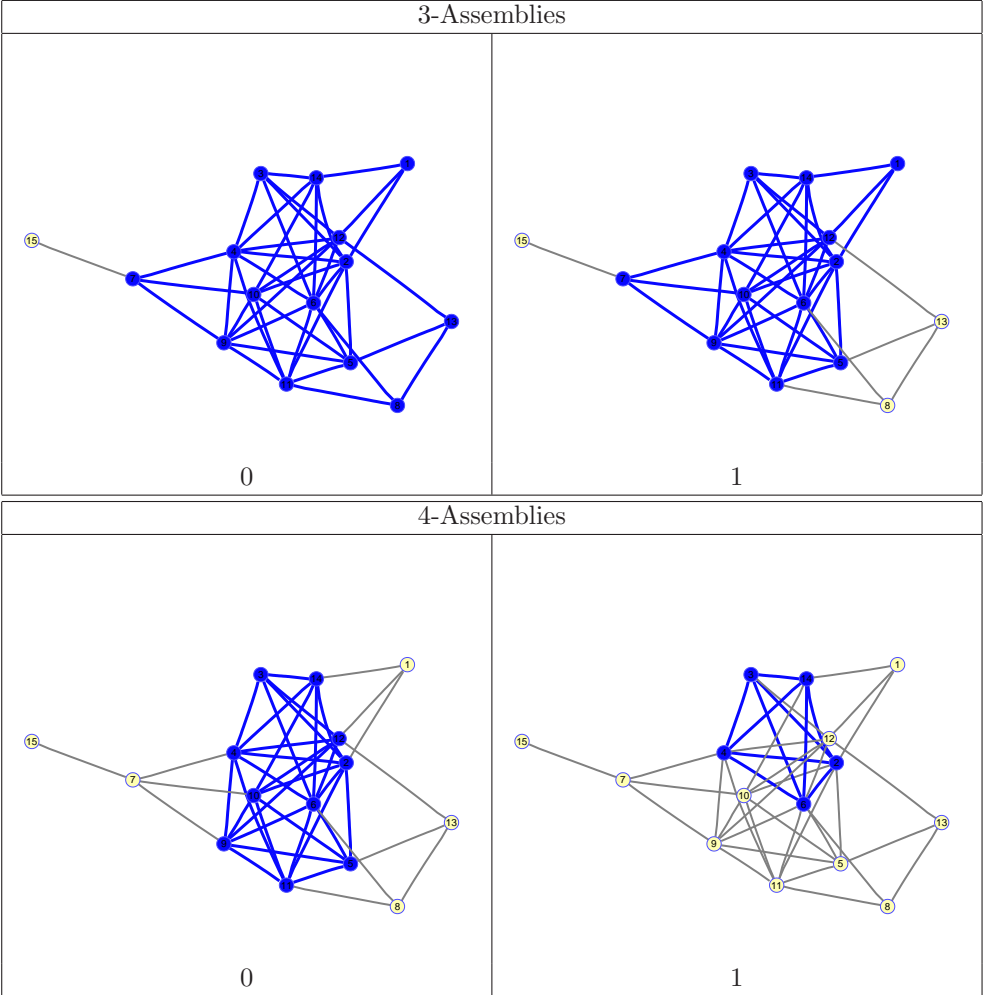
0

B.2.31 `randomGraph(15,0.25,undirected)-3`

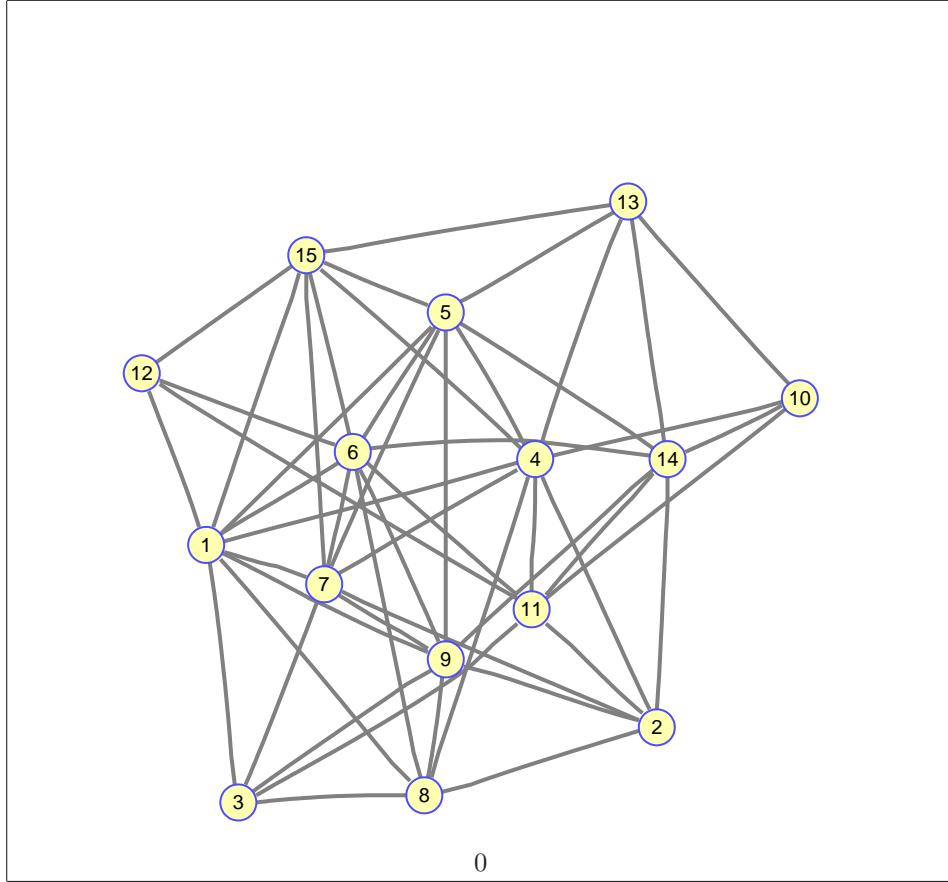


0	1	0	0	0	0	0	0	0	0	0	1	0	1	0
1	0	1	1	1	1	0	0	0	1	1	0	0	1	0
0	1	0	1	0	1	0	0	0	0	0	1	0	1	0
0	1	1	0	0	1	1	0	1	0	1	1	0	1	0
0	1	0	0	0	1	0	0	1	1	1	0	1	0	0
0	1	1	1	1	0	0	1	1	0	1	1	0	1	0
0	0	0	1	0	0	0	0	1	1	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	1	0	1	0	0
0	0	0	1	1	1	1	0	0	1	1	1	0	0	0
0	1	0	0	1	0	1	0	1	0	1	1	0	1	0
0	1	0	1	1	1	0	1	1	1	0	0	0	0	0
1	0	1	1	0	1	0	0	1	1	0	0	1	0	0
0	0	0	0	1	0	0	1	0	0	0	1	0	0	0
1	1	1	1	0	1	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

(31)



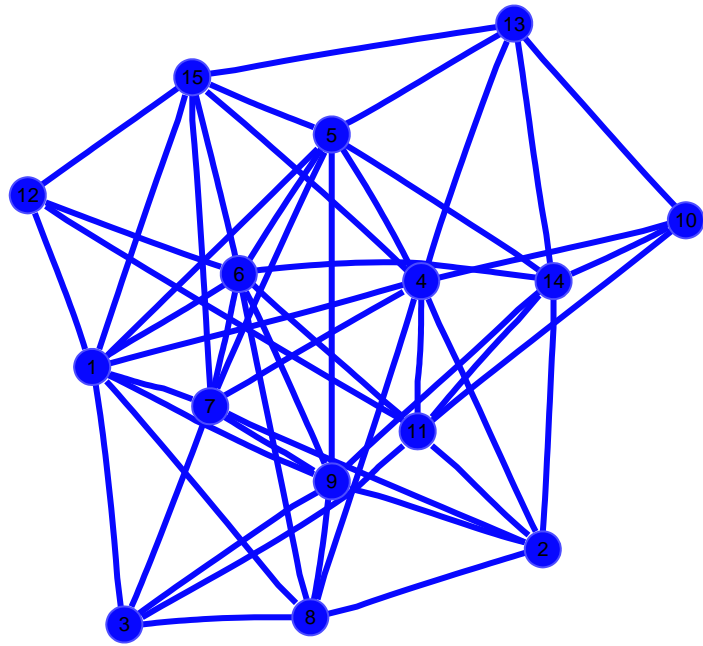
B.2.32 randomGraph(15,0.325,undirected)-1



0	0	1	1	1	1	1	1	1	1	0	0	1	0	0	1
0	0	0	1	0	0	1	1	1	1	0	1	0	0	1	0
1	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0
1	1	0	0	1	0	1	1	0	1	1	0	1	0	1	0
1	0	0	1	0	1	1	0	1	0	0	0	1	1	1	1
1	0	0	0	1	0	1	1	1	0	1	1	0	1	1	1
1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	1
1	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0
1	1	1	0	1	1	1	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0
0	1	1	1	0	1	0	0	0	1	0	1	0	1	0	0
1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1
0	0	0	1	1	0	0	0	0	1	0	0	0	1	1	1
0	1	0	0	1	1	0	0	1	1	1	0	1	0	0	0
1	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0

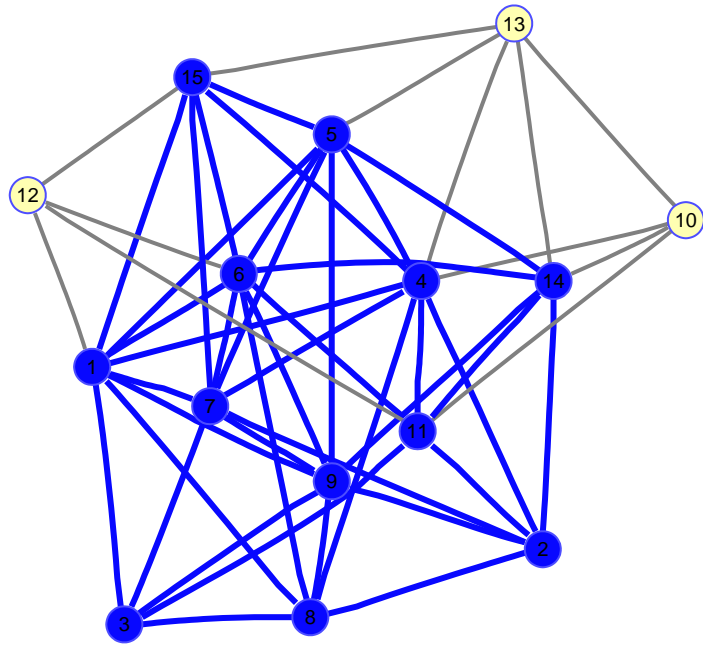
(32)

4-Assemblies



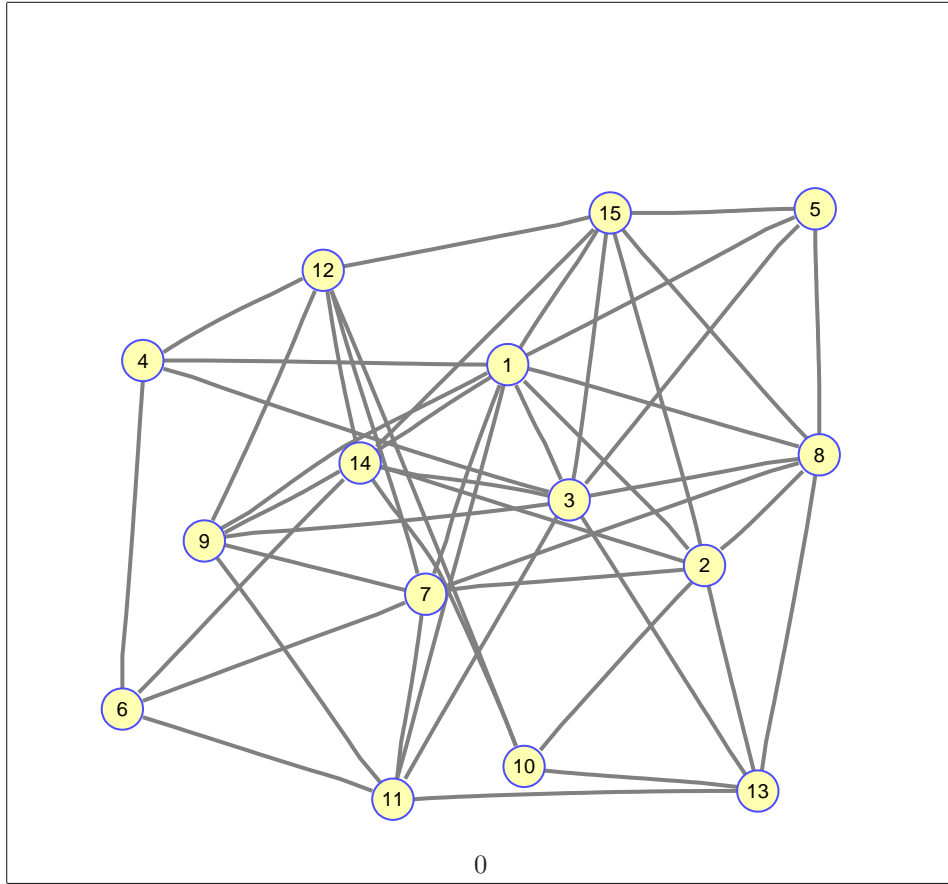
0

5-Assemblies



0

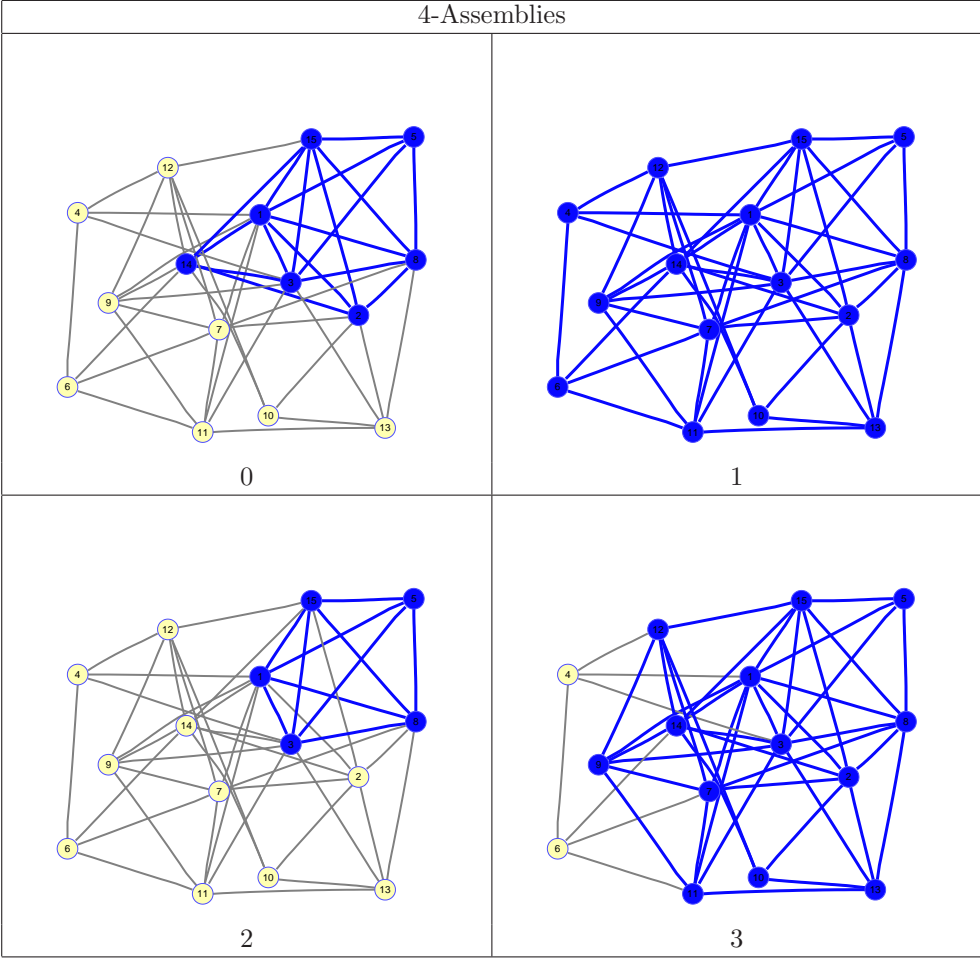
B.2.33 randomGraph(15,0.325,undirected)-2



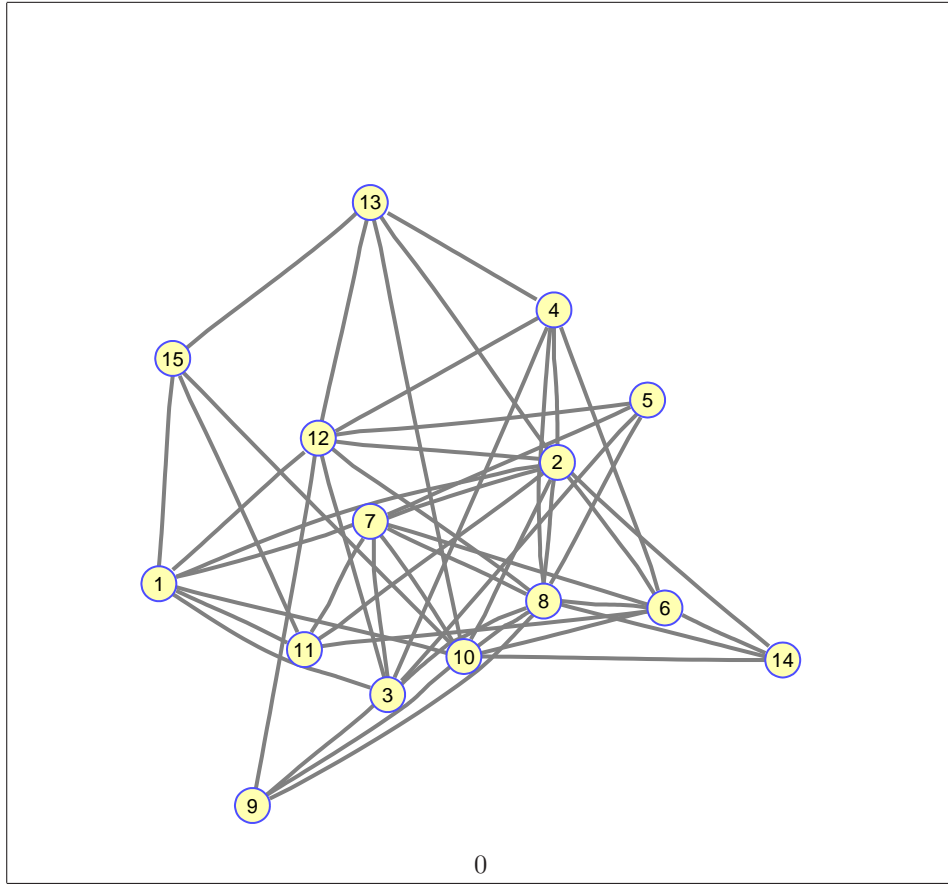
0	1	1	1	1	0	1	1	1	0	1	0	0	1	1
1	0	0	0	0	0	1	1	0	1	0	0	1	1	1
1	0	0	1	1	0	0	1	1	0	1	0	1	1	1
1	0	1	0	0	1	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	1	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	0	0	1	0
1	1	0	0	0	1	0	1	1	0	1	1	0	0	0
1	1	1	0	1	0	1	0	0	0	0	0	1	0	1
1	0	1	0	0	0	1	0	0	0	1	1	0	1	0
0	1	0	0	0	0	0	0	0	0	0	1	1	1	0
1	0	1	0	0	1	1	0	1	0	0	0	1	0	0
0	0	0	1	0	0	1	0	1	1	0	0	0	1	1
0	1	1	0	0	0	0	1	0	1	1	0	0	0	0
1	1	1	0	0	1	0	0	1	1	0	1	0	0	1
1	1	1	0	1	0	0	1	0	0	0	1	0	1	0

(33)

4-Assemblies



B.2.34 randomGraph(15,0.325,undirected)-3

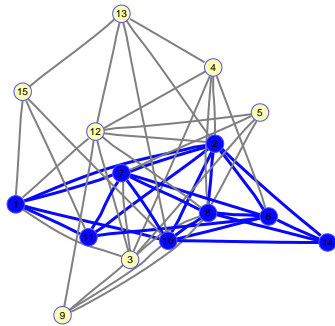


0

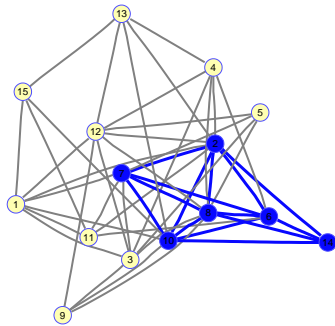
0	1	1	0	0	0	1	0	0	1	1	1	0	0	1
1	0	0	1	0	1	1	1	0	1	1	1	1	1	0
1	0	0	1	1	0	1	1	1	0	0	1	0	0	0
0	1	1	0	0	1	0	1	0	0	0	1	1	0	0
0	0	1	0	0	0	1	1	0	0	0	1	0	0	0
0	1	0	1	0	0	1	1	0	1	1	0	0	1	0
1	1	1	0	1	1	0	1	0	1	1	0	0	0	0
0	1	1	1	1	1	1	0	1	1	0	1	0	1	0
0	0	1	0	0	0	0	1	0	1	0	1	0	0	0
1	1	0	0	0	1	1	1	1	0	0	0	1	1	1
1	1	0	0	0	1	1	0	0	0	0	0	0	0	1
1	1	1	1	1	0	0	1	1	0	0	0	1	0	0
0	1	0	1	0	0	0	0	0	1	0	1	0	0	1
0	1	0	0	0	1	0	1	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0	0	1	1	0	1	0	0

(34)

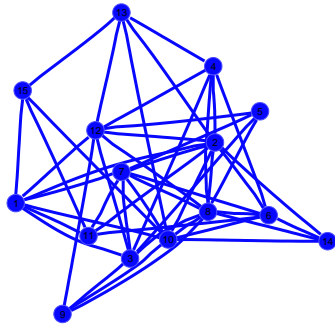
4-Assemblies



0

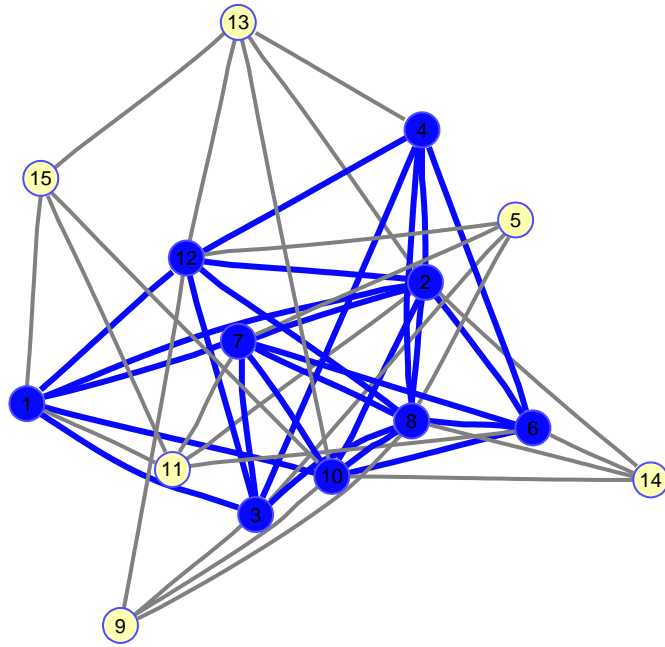


1



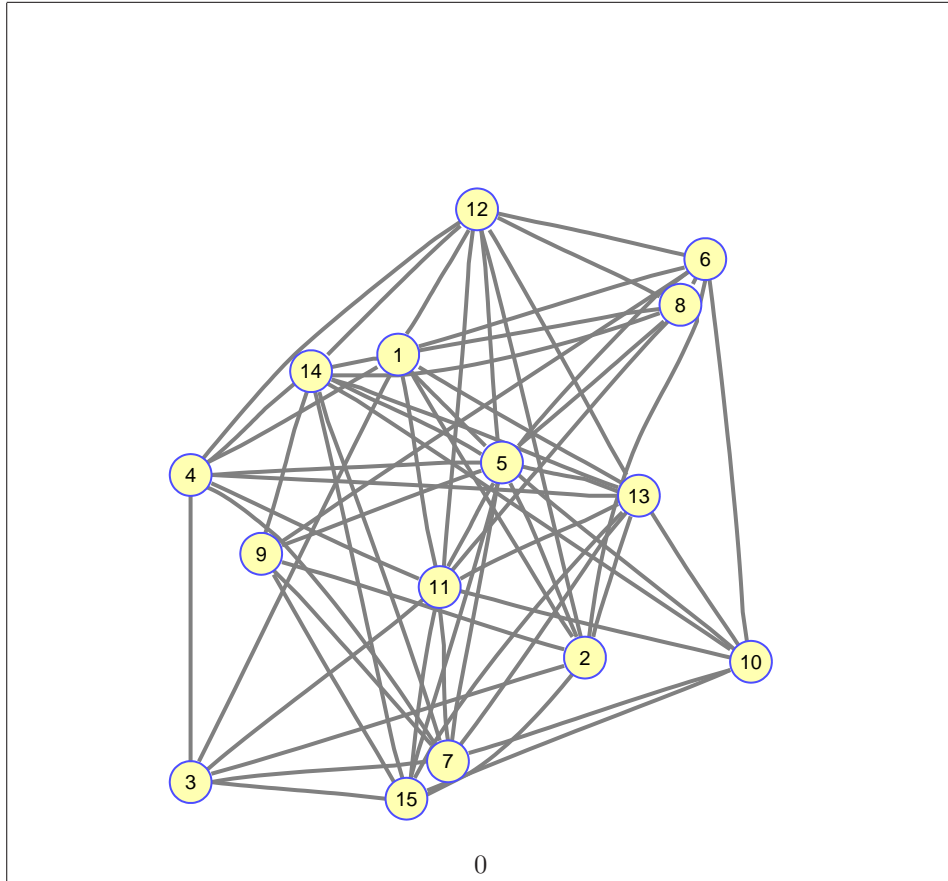
2

5-Assemblies



0

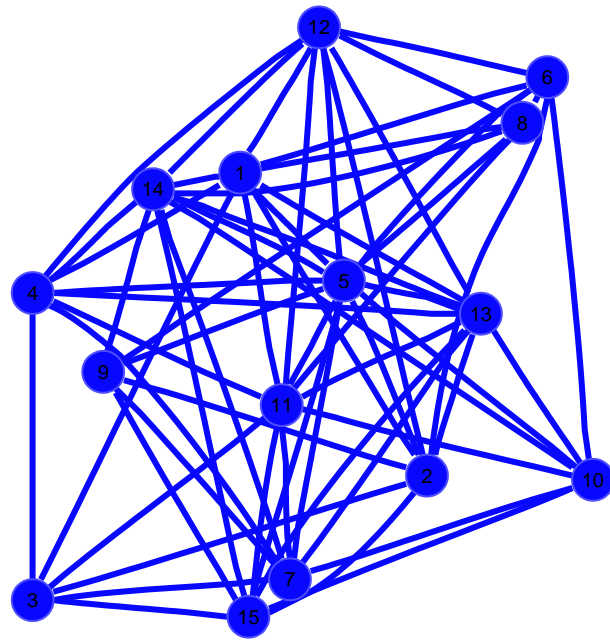
B.2.35 randomGraph(15,0.5,undirected)-1



0	1	1	1	1	1	0	1	0	0	1	1	1	1	0
1	0	1	0	1	1	0	0	1	0	0	1	1	0	1
1	1	0	1	0	0	1	0	0	0	1	0	0	0	1
1	0	1	0	1	0	1	0	0	0	1	1	1	1	0
1	1	0	1	0	1	1	1	1	1	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0	1	0	0	0
0	0	1	1	1	0	0	0	1	1	1	0	1	1	0
1	0	0	0	1	1	0	0	0	0	1	1	0	1	0
0	1	0	0	1	1	1	0	0	0	0	0	0	1	1
0	0	0	0	1	1	1	0	0	0	1	0	1	1	1
1	0	1	1	1	0	1	1	0	1	0	1	1	0	1
1	1	0	1	1	1	0	1	0	0	1	0	1	1	0
1	1	0	1	1	0	1	0	0	1	1	1	0	1	1
1	0	0	1	1	0	1	1	1	1	0	1	1	0	1
0	1	1	0	1	0	0	0	1	1	1	0	1	1	0

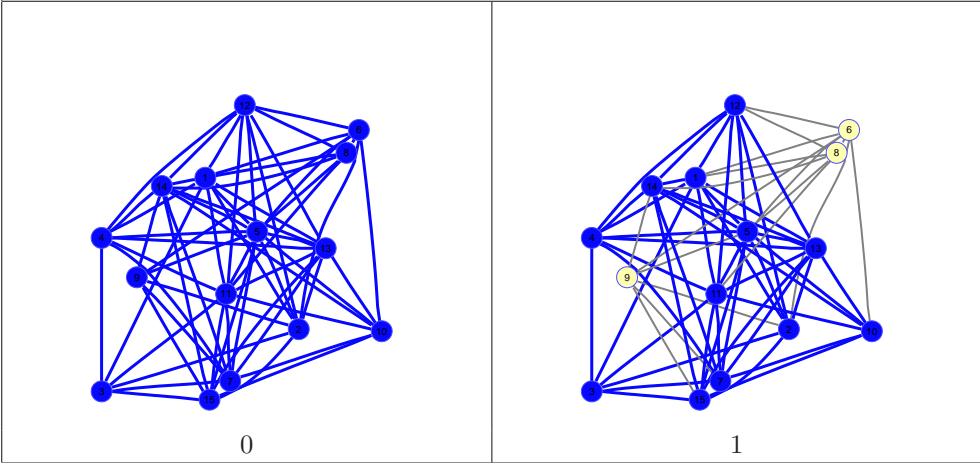
(35)

5-Assemblies

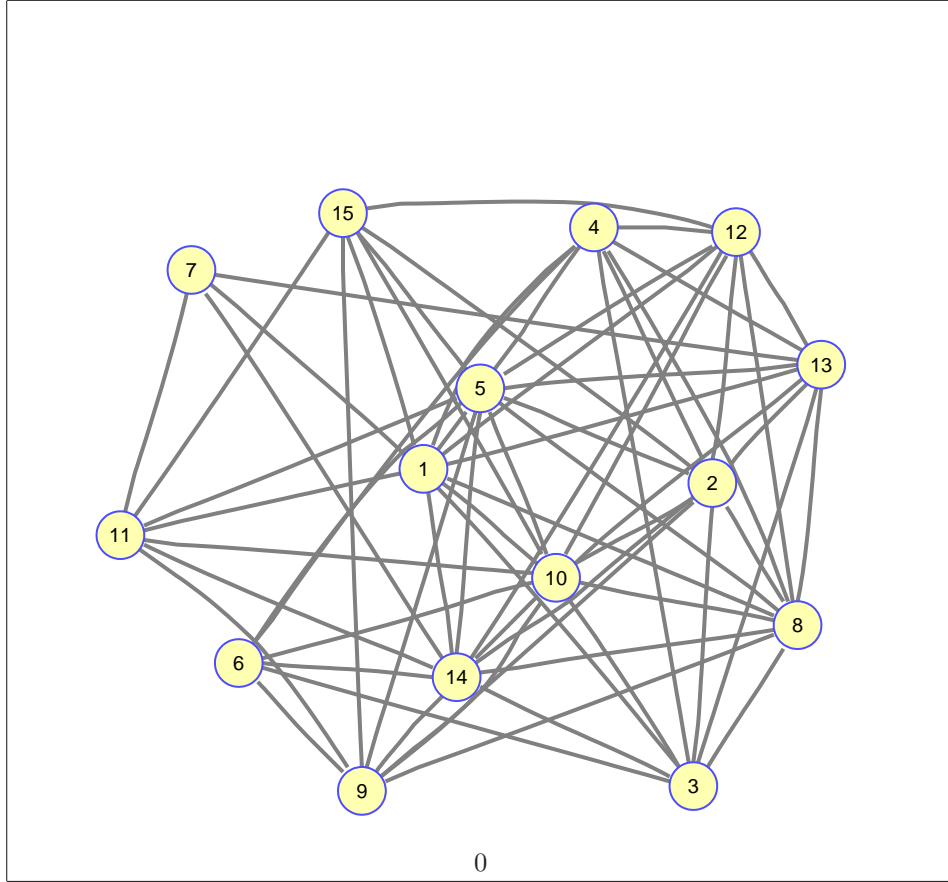


0

6-Assemblies



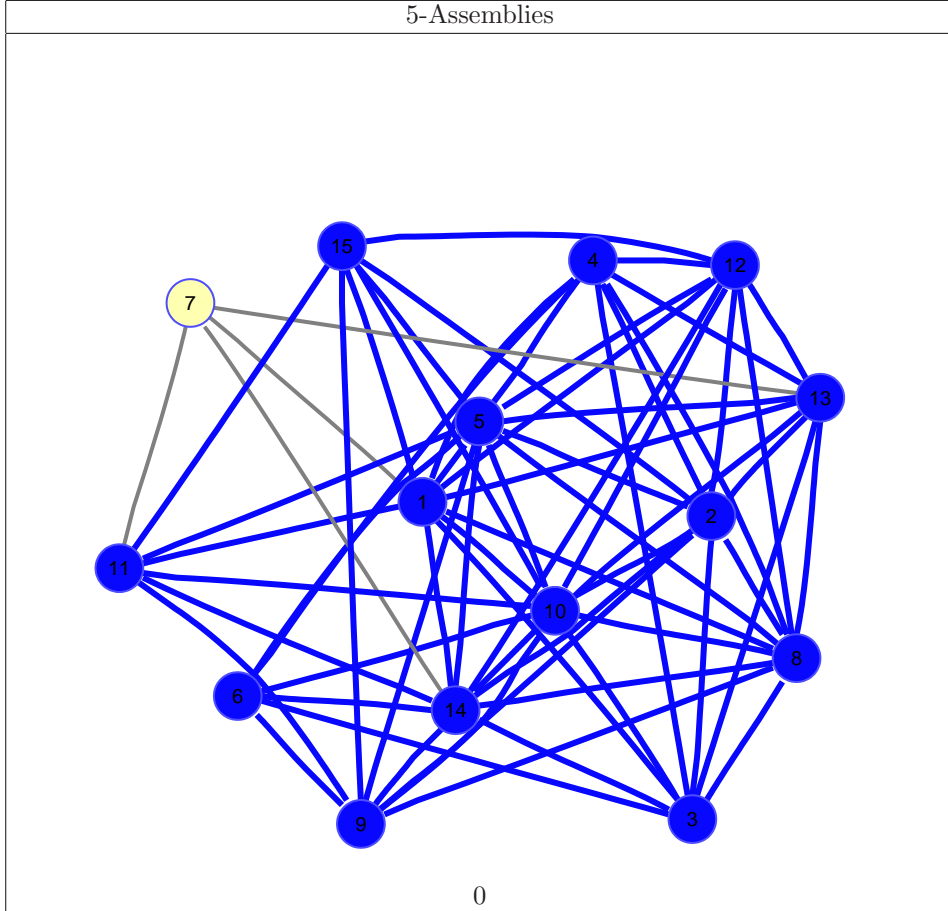
B.2.36 randomGraph(15,0.5,undirected)-2



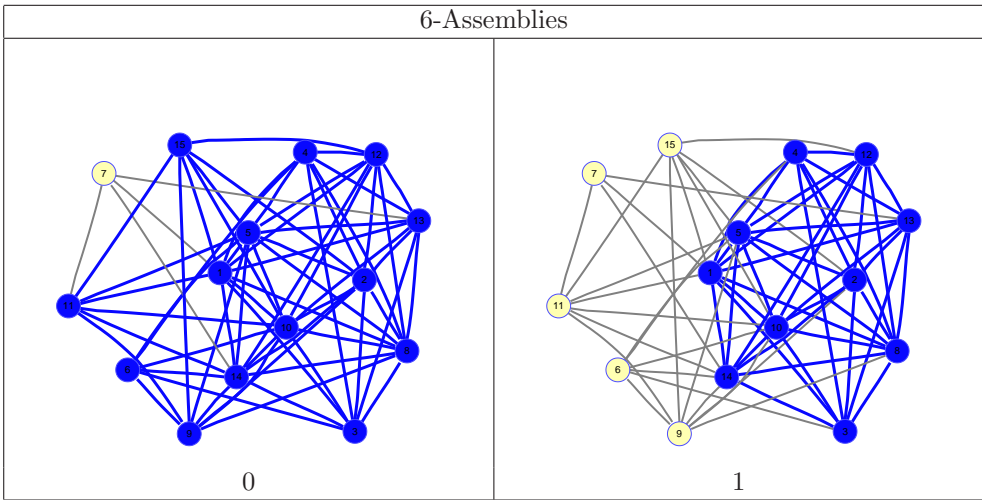
0	0	1	1	1	0	1	1	0	1	1	1	1	1	1
0	0	1	1	1	0	0	1	1	1	0	1	1	1	1
1	1	0	1	0	1	0	1	0	1	0	0	1	1	0
1	1	1	0	1	1	0	1	0	0	0	1	1	0	0
1	1	0	1	0	1	0	1	1	1	1	1	1	1	1
0	0	1	1	1	0	0	0	1	1	0	0	0	1	0
1	0	0	0	0	0	0	0	0	0	1	0	1	1	0
1	1	1	1	1	0	0	0	1	1	0	1	1	1	0
0	1	0	0	1	1	0	1	0	1	1	0	0	1	1
1	1	1	0	1	1	0	1	1	0	1	1	1	1	1
1	0	0	0	1	0	1	0	1	1	0	0	0	1	1
1	1	0	1	1	0	0	1	0	1	0	0	1	1	1
1	1	1	1	1	0	1	1	0	1	0	1	0	0	0
1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
1	1	0	0	1	0	0	0	1	1	1	1	0	0	0

(36)

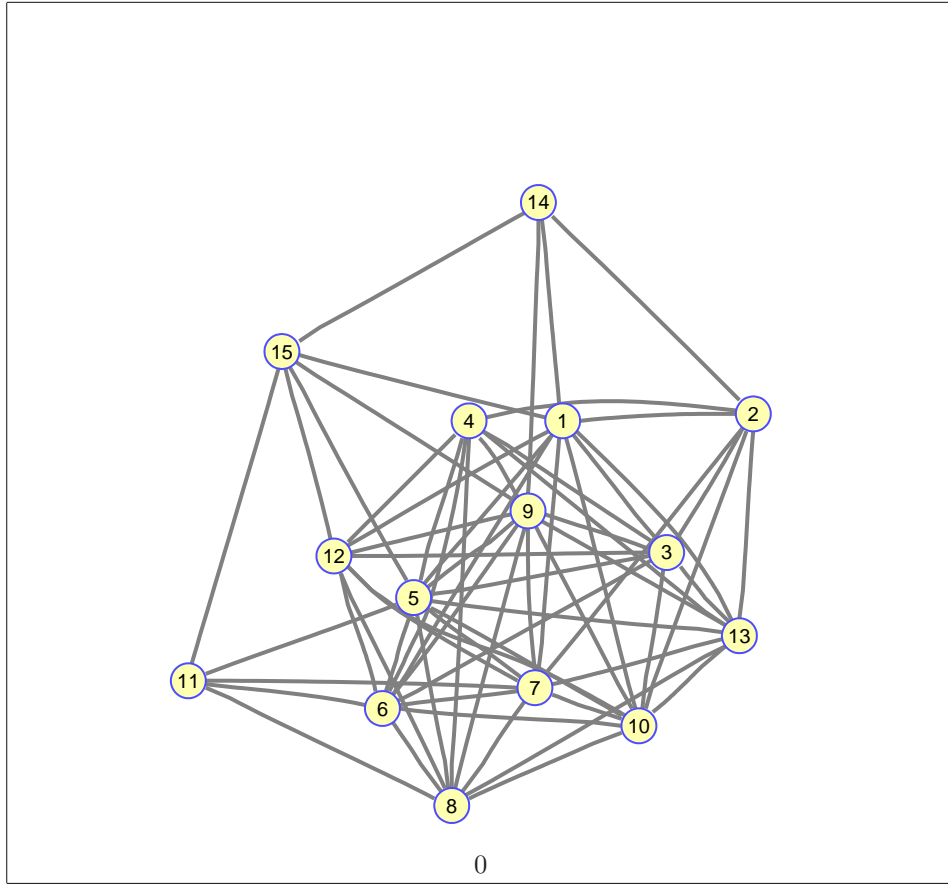
5-Assemblies



6-Assemblies



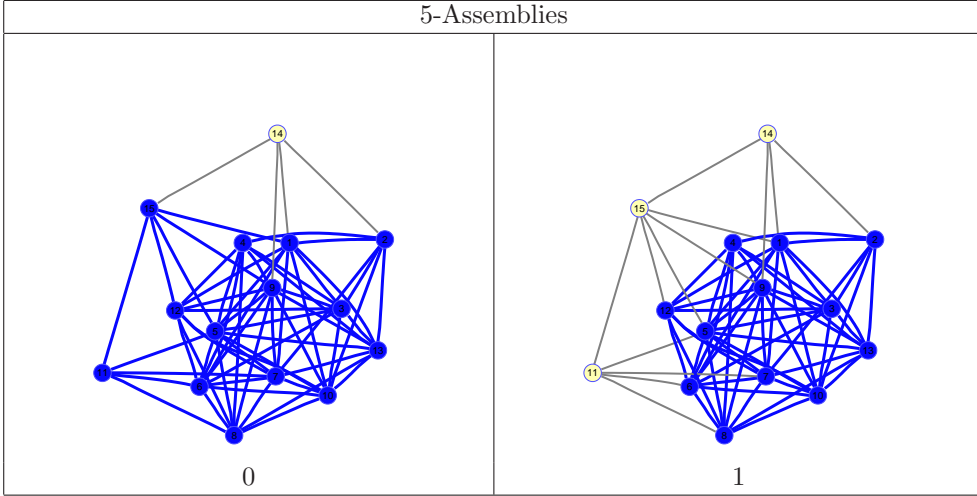
B.2.37 randomGraph(15,0.5,undirected)-3



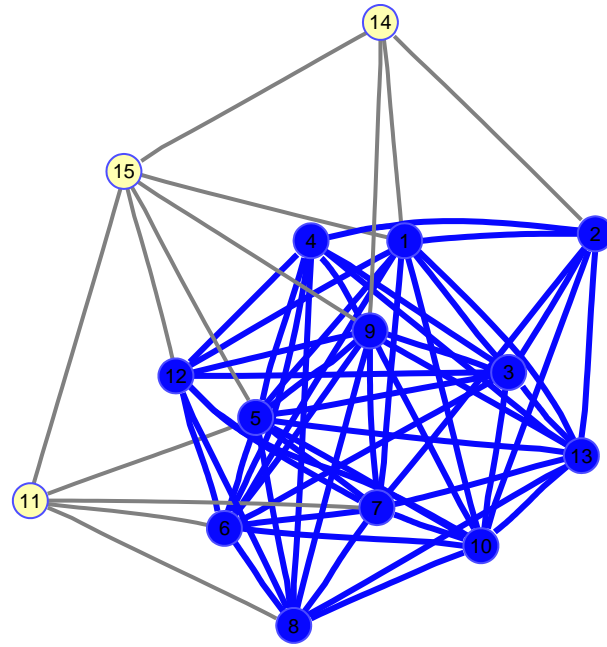
0	1	1	0	1	1	1	0	0	1	0	1	1	1	1
1	0	1	1	0	0	1	0	0	1	0	0	1	1	0
1	1	0	1	1	1	0	0	1	1	0	1	1	0	0
0	1	1	0	1	1	0	1	1	0	0	1	1	0	0
1	0	1	1	0	1	1	1	1	1	1	0	1	0	1
1	0	1	1	1	0	1	1	1	1	1	1	0	0	0
1	1	0	0	1	1	0	1	1	1	1	1	1	0	0
0	0	0	1	1	1	1	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	1	0	1	1	1	1
1	1	1	0	1	1	1	1	1	0	0	1	1	0	0
0	0	0	0	1	1	1	1	0	0	0	0	0	0	1
1	0	1	1	0	1	1	1	1	1	0	0	0	0	1
1	1	1	1	1	0	1	1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0	1	0	0	0	0	0	1
1	0	0	0	1	0	0	0	1	0	1	1	0	1	0

(37)

5-Assemblies

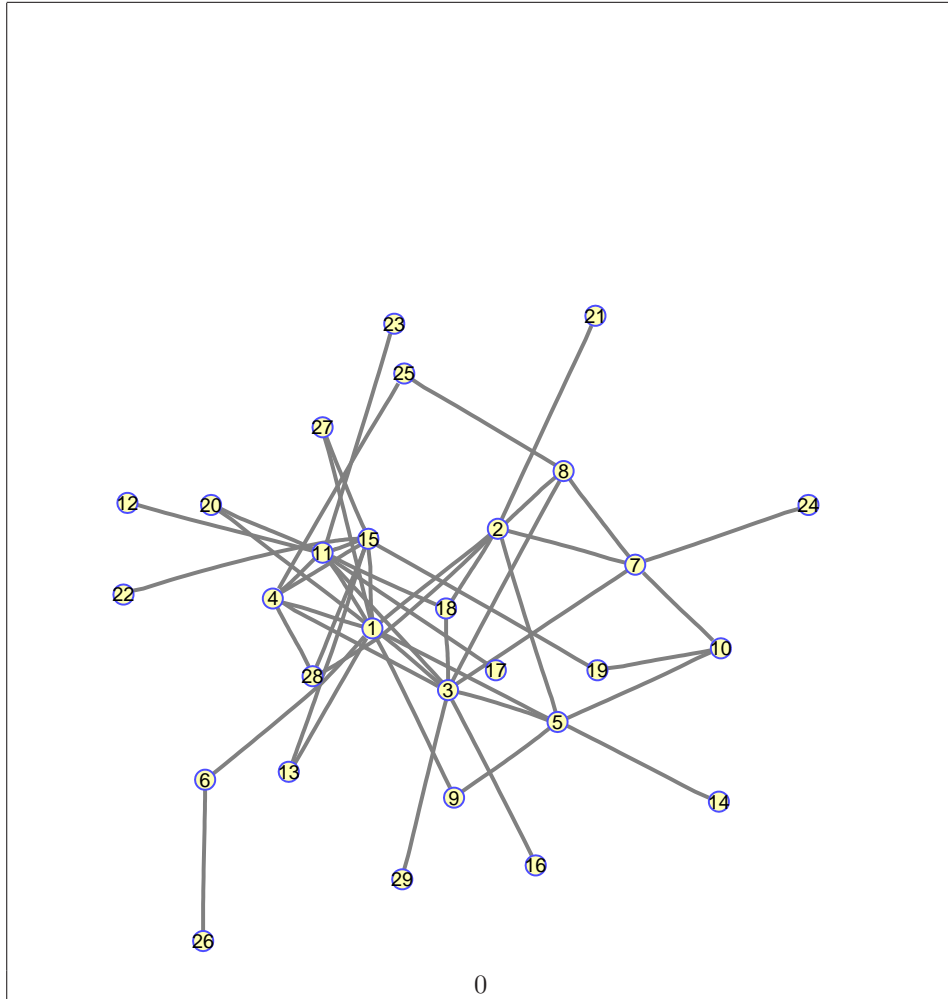


6-Assemblies



0

B.2.38 scaleFreeGraph(50,0.5,0,0,1,[0.8,0.2],[0.8,0.2],undirected)-1



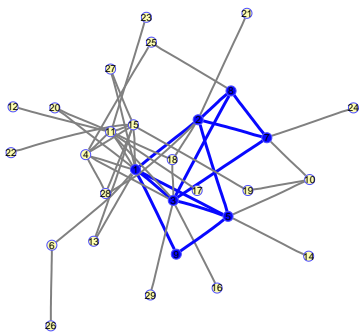
```

2 3 2 1 2 1 0 0 2 0 1 0 1 0 1 0 0 0 0 1 0 0 0 0 0 0 1 0 0
3 0 0 0 2 0 1 1 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 1 0
2 0 1 1 1 0 3 1 0 0 1 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1
1 0 1 1 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 2 0
2 2 1 0 0 0 0 0 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 1 3 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
2 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 0 1 1 0 0 0 0 0 0 0 0 2 0 0 1 0 1 1 0 1 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 1 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 1 1 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 2 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

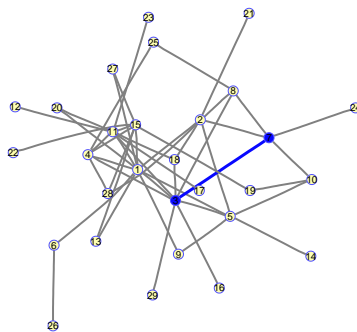
```

(38)

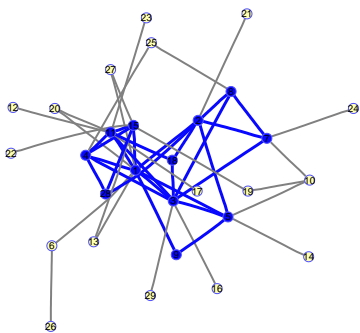
3-Assemblies



0

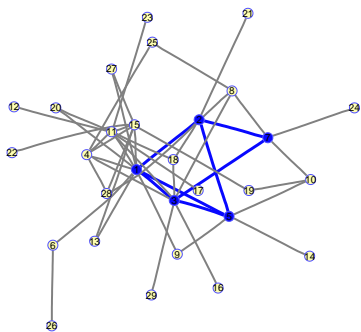


1

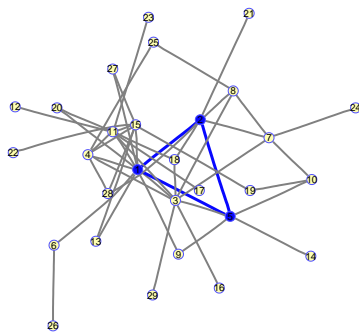


2

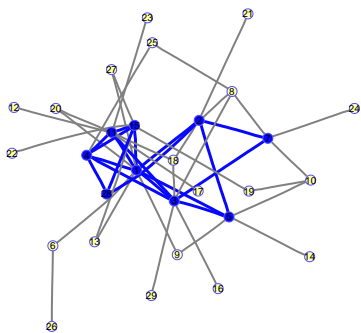
4-Assemblies



0

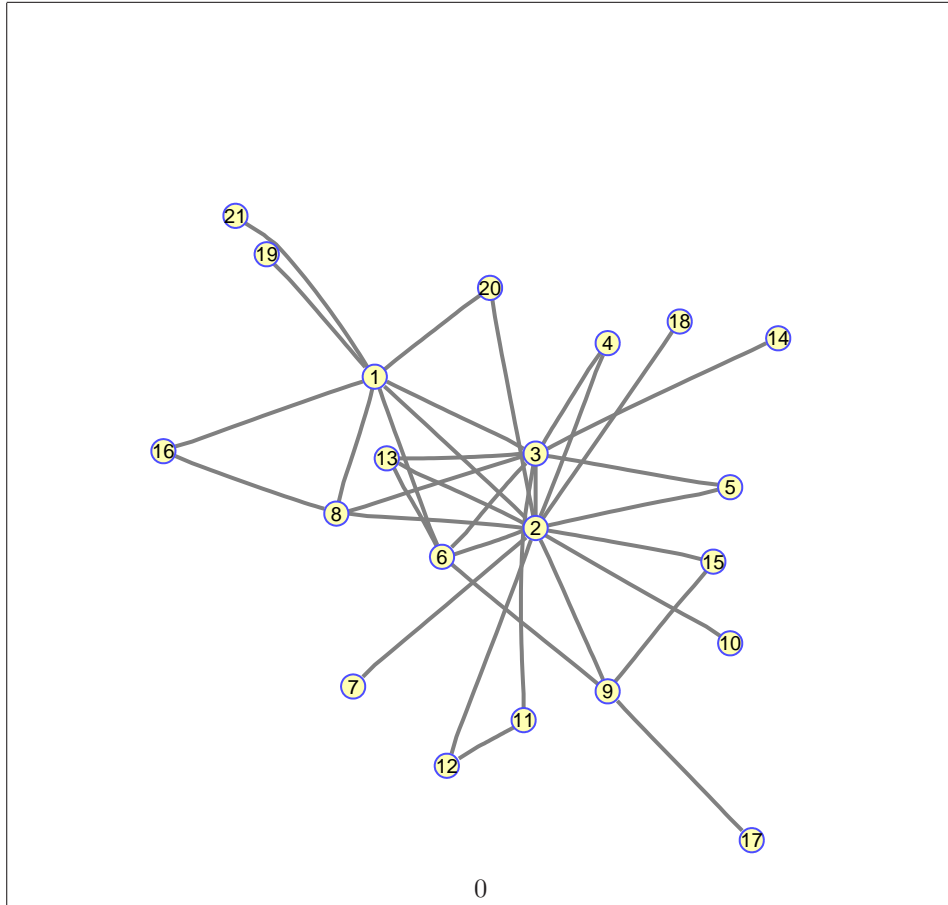


1



2

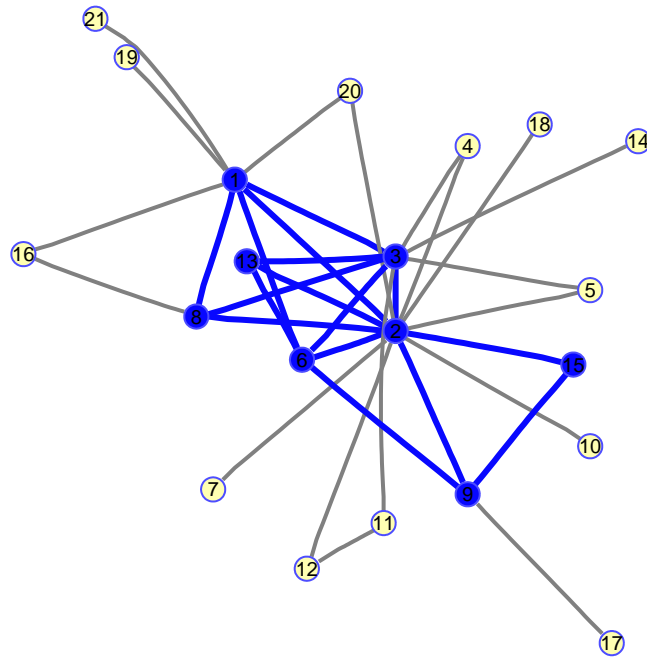
B.2.39 scaleFreeGraph(50,0.5,0,0,1,[0.8,0.2],[0.8,0.2],undirected)-2



0	6	3	0	0	2	0	1	0	0	0	0	0	0	1	0	0	1	1	1	
6	5	10	1	1	2	1	1	3	2	0	1	1	0	2	0	0	1	0	1	0
3	10	2	1	1	1	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	0	2	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	3	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

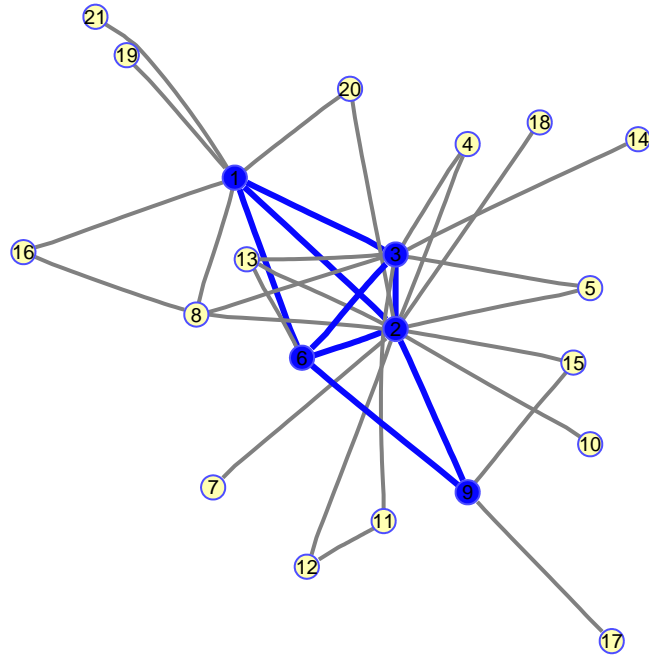
(39)

3-Assemblies



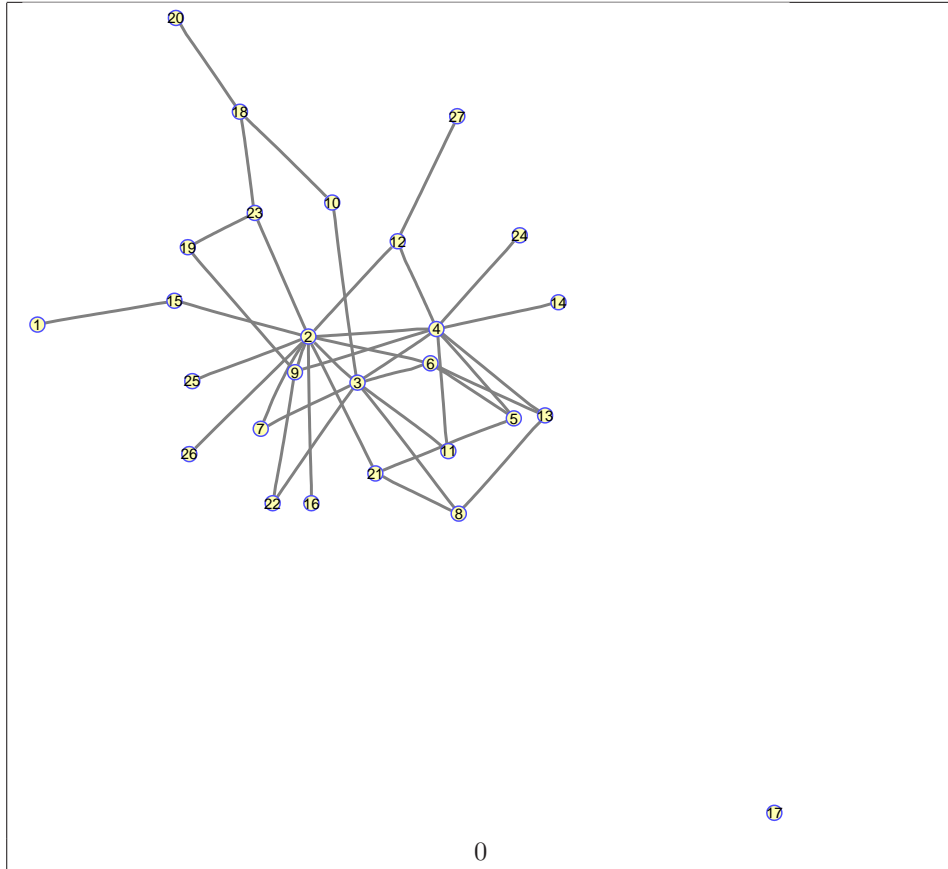
0

4-Assemblies



0

B.2.40 scaleFreeGraph(50,0.5,0,0,1,[0.8,0.2],[0.8,0.2],undirected)-3

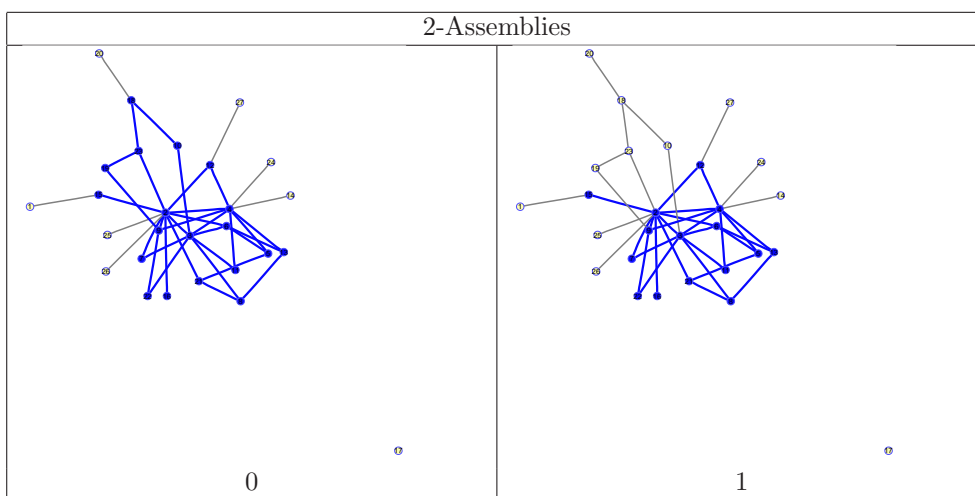


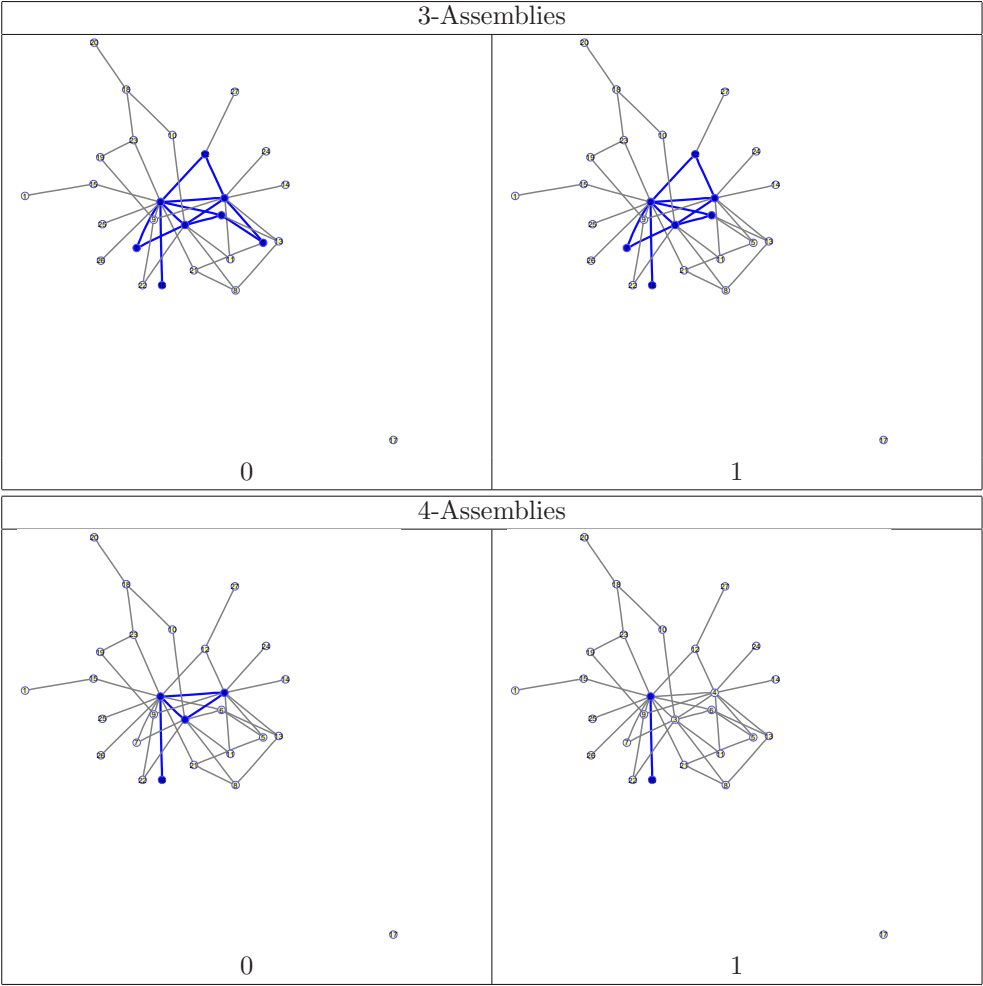
```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 2 3 2 0 1 1 0 1 0 0 1 0 0 2 4 0 0 0 0 1 0 1 0 1 1 0
0 3 1 2 0 2 2 2 0 1 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 2 2 1 1 0 0 0 1 0 1 2 1 1 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 1 2 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 2 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
0 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 1 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0

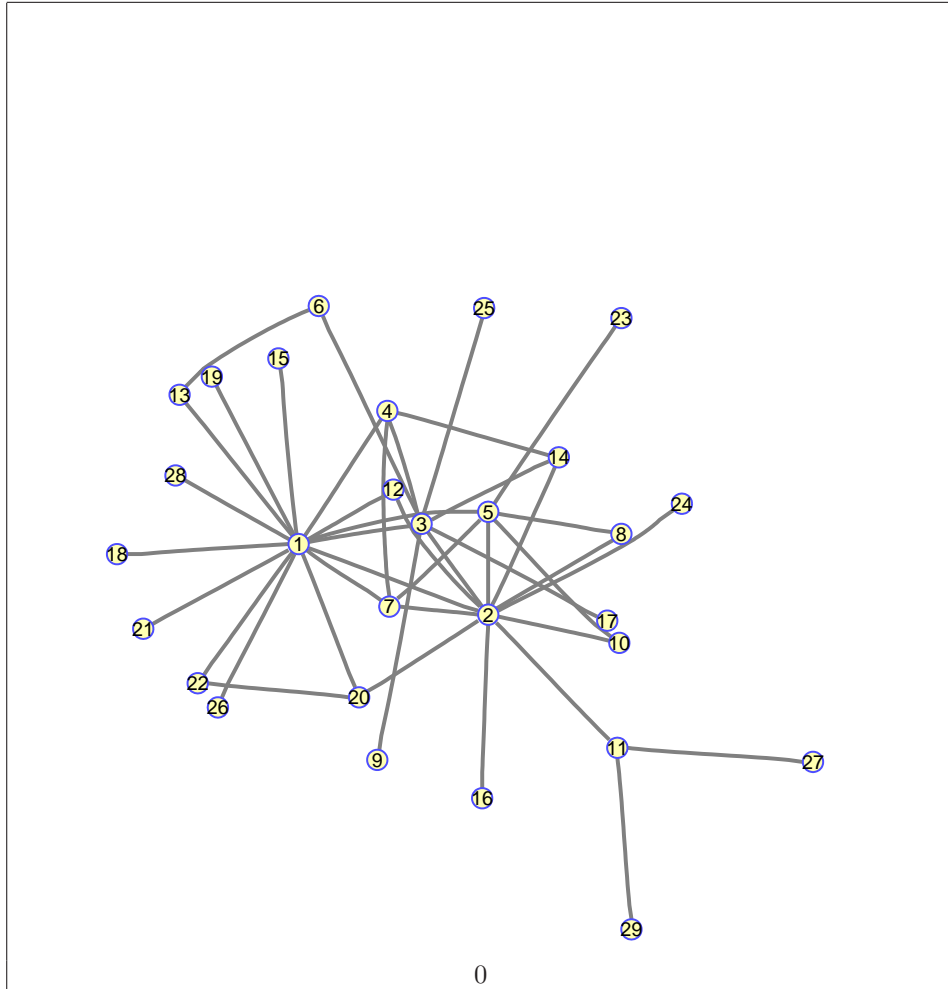
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(40)





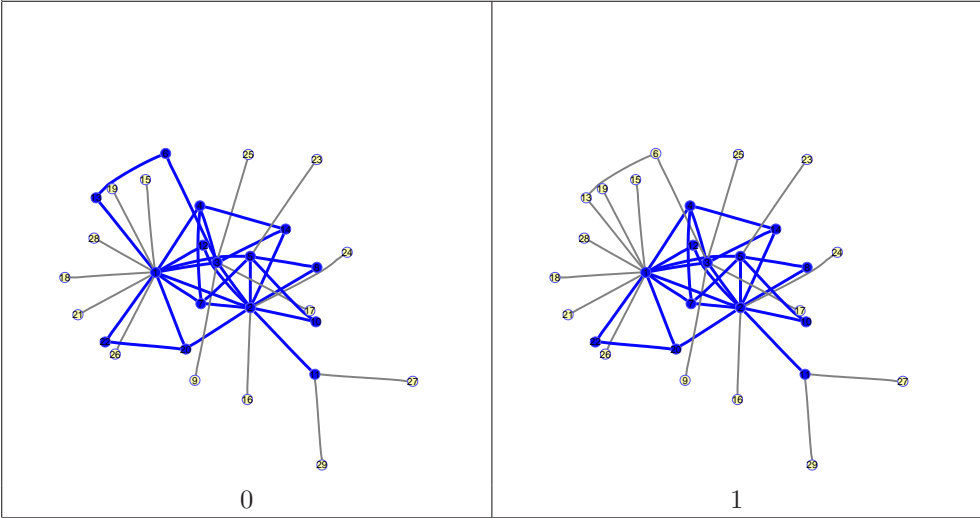
B.2.41 scaleFreeGraph(60,0.5,0,0,1,[1],[1],undirected)-1



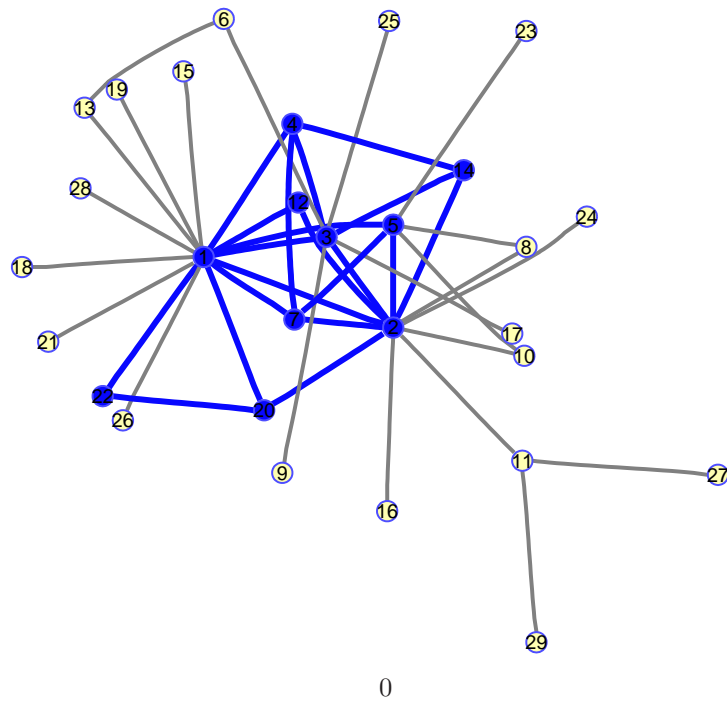
4	3	4	1	1	0	1	0	0	0	0	2	1	0	1	0	0	1	1	1	1	3	0	0	0	1	0	1	0			
3	1	2	0	1	0	1	1	0	1	2	1	0	1	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0			
4	2	0	3	0	1	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0			
1	0	3	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
1	1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0			
0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1		
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	
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3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	
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0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
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0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(41)

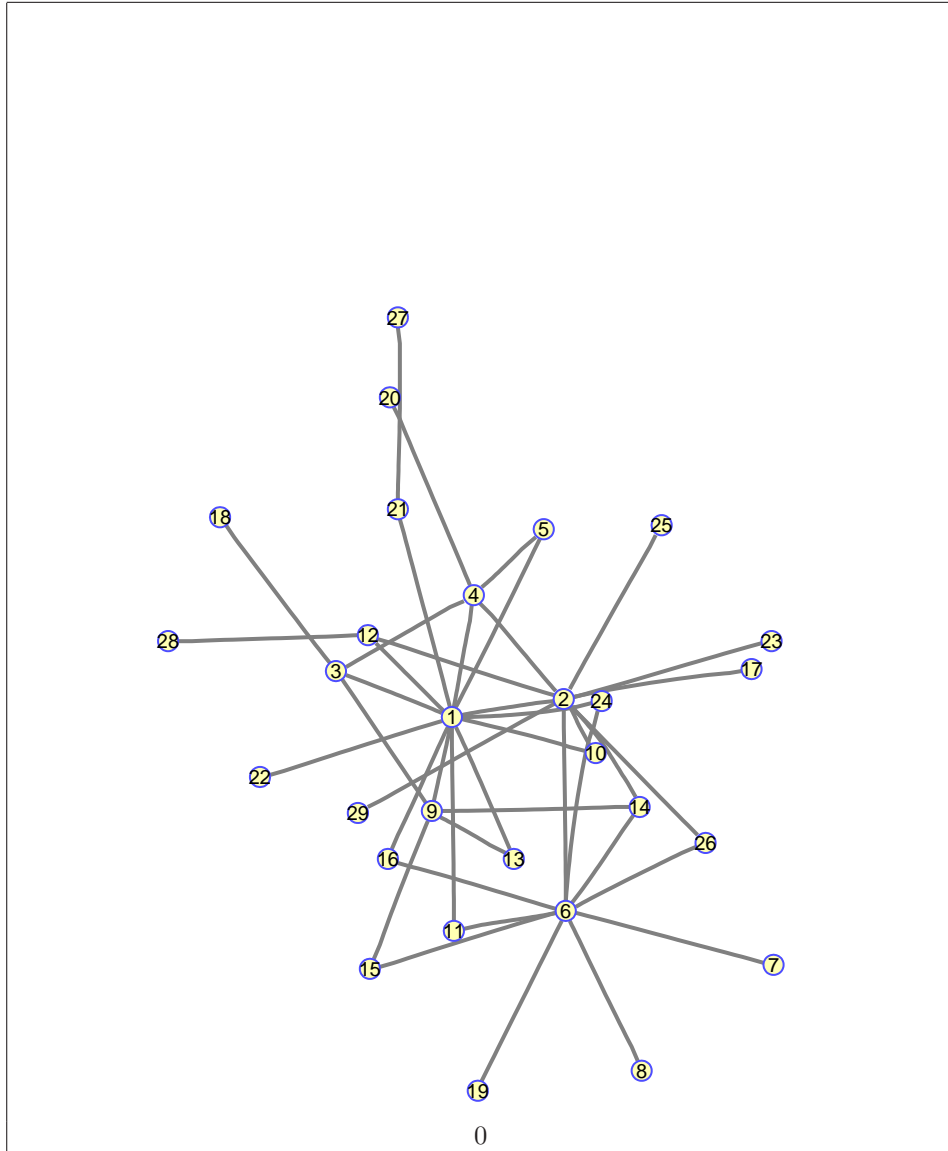
2-Assemblies



3-Assemblies



B.2.42 scaleFreeGraph(60,0.5,0,0,1,[1],[1],undirected)-2



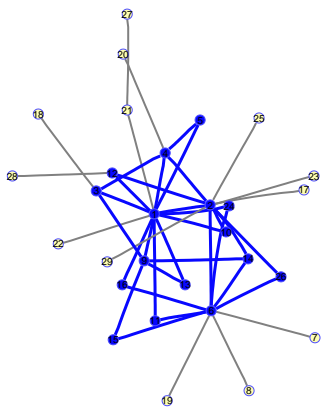

```

3 5 4 2 3 0 0 0 1 1 1 2 2 0 0 1 0 0 0 0 1 1 0 1 0 0 0 0
5 0 0 2 0 1 0 0 0 1 0 1 0 1 0 0 1 0 0 0 0 0 1 0 1 1 0 0 1
4 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
2 2 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
3 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 1 1 0 0 2 0 0 1 1 1 0 0 1 0 0 0 0 1 0 1 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 0 0 0 0 1 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
2 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
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1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

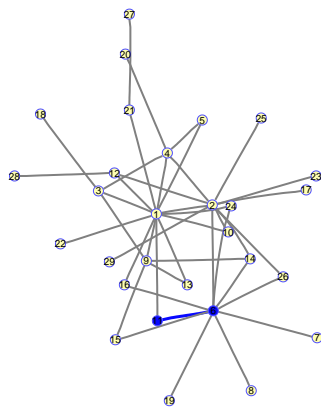
```

(42)

2-Assemblies

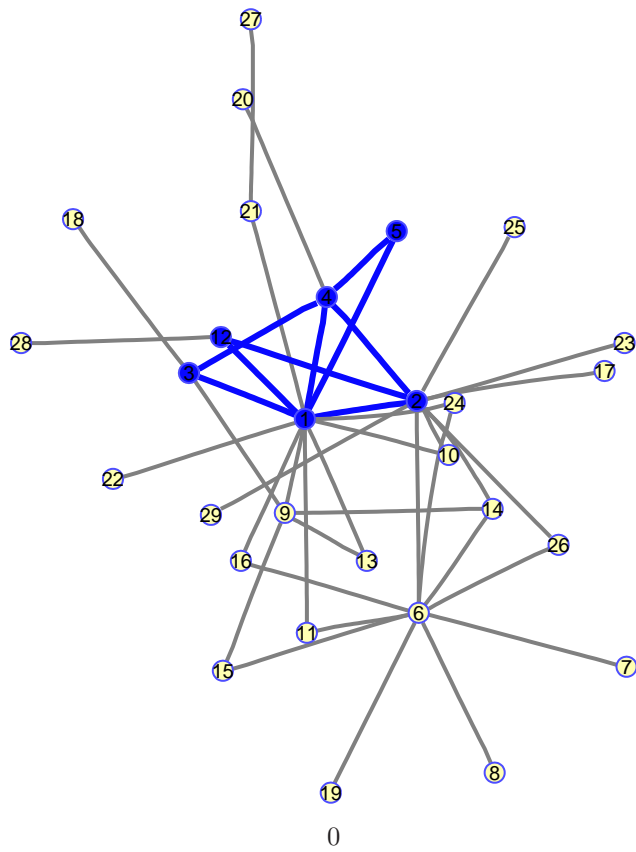


0

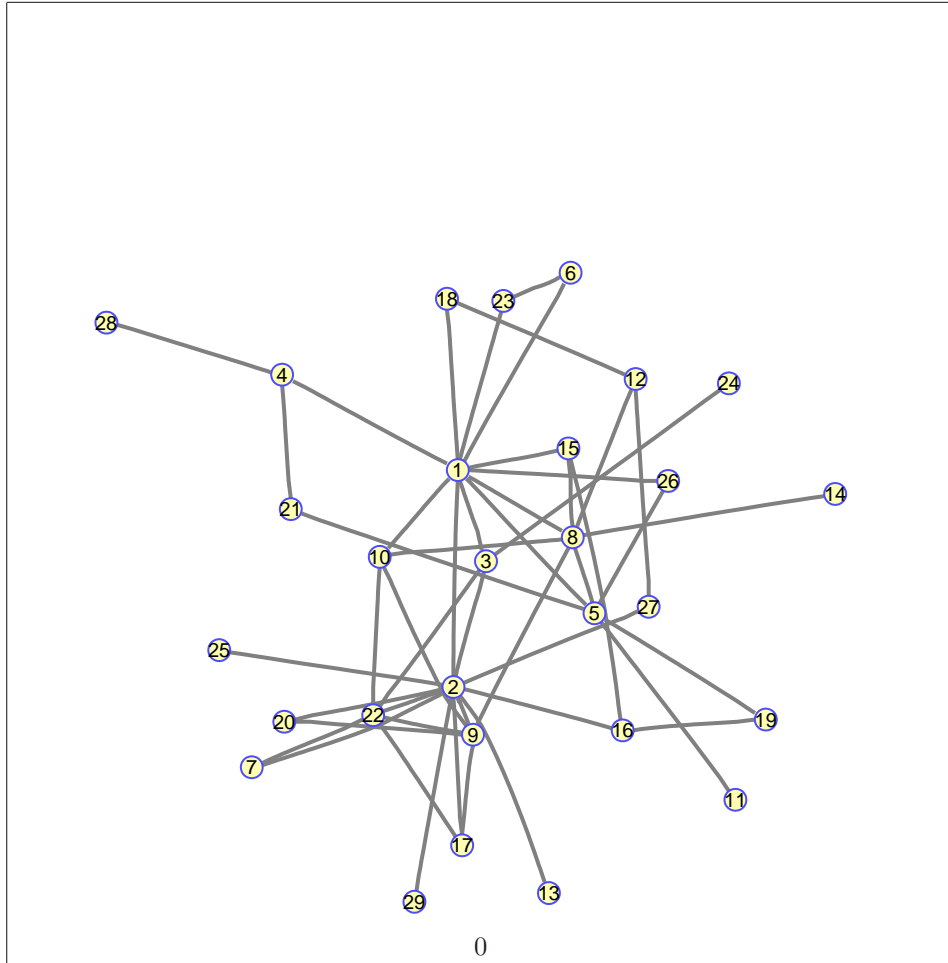


1

3-Assemblies



B.2.43 scaleFreeGraph(60,0.5,0,0,1,[1],[1],undirected)-3



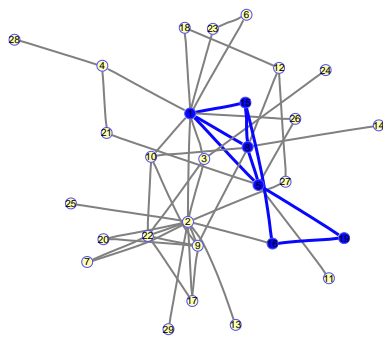
```

0 1 1 1 1 1 0 1 0 1 0 0 0 0 1 0 0 1 0 0 0 0 2 0 0 1 0 0 0
1 3 2 0 0 0 3 0 1 0 0 0 1 0 0 1 1 0 0 1 0 1 0 0 2 0 1 0 1
1 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0
1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 1 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1 0 0 0 1 0 0 0 1 1 0 2 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 1 0 0 1 0 1 0 0 0 0 0 0
1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 2 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 1 0 0 0 1 0 1 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
2 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

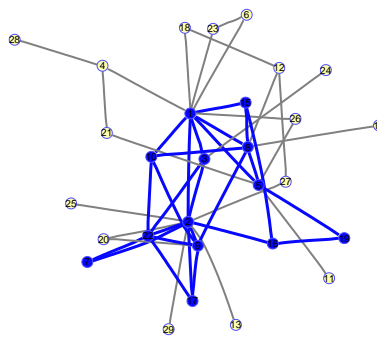
```

(43)

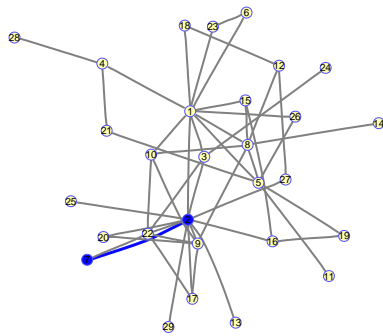
3-Assemblies



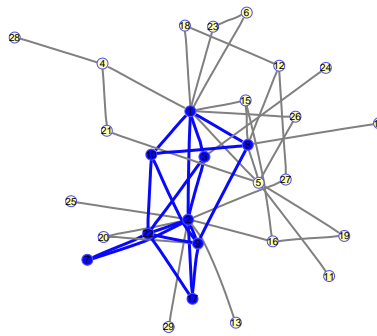
0



1

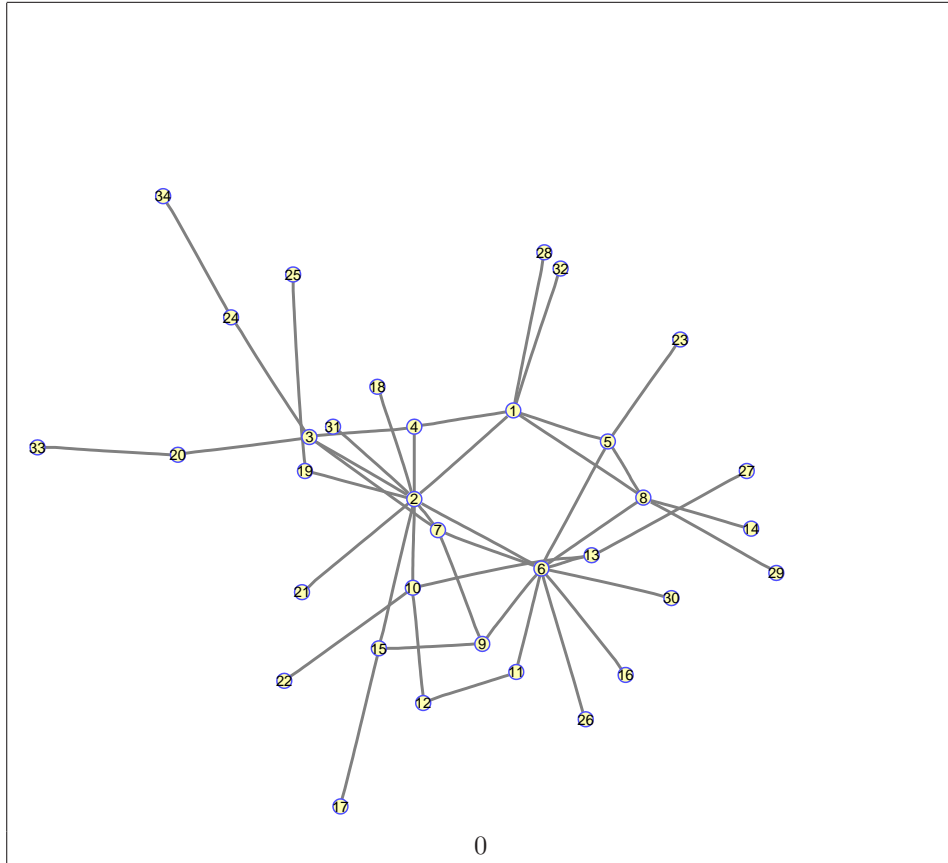


2

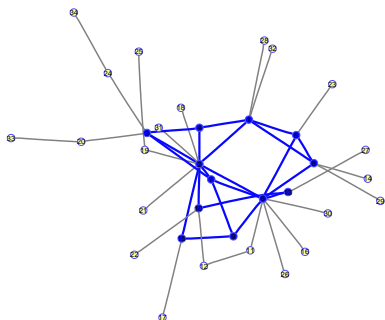


3

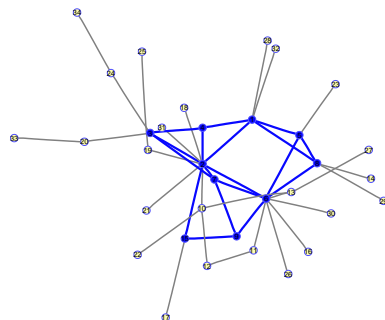
B.2.44 scaleFreeGraph(60,0.5,0.5,0.5,0.5,[1],[1],undirected)-2



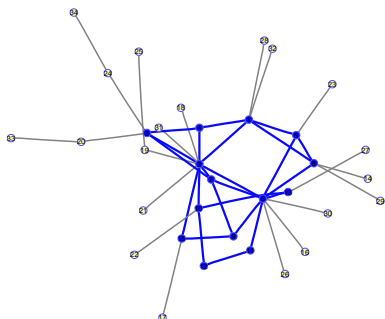
2-Assemblies



0

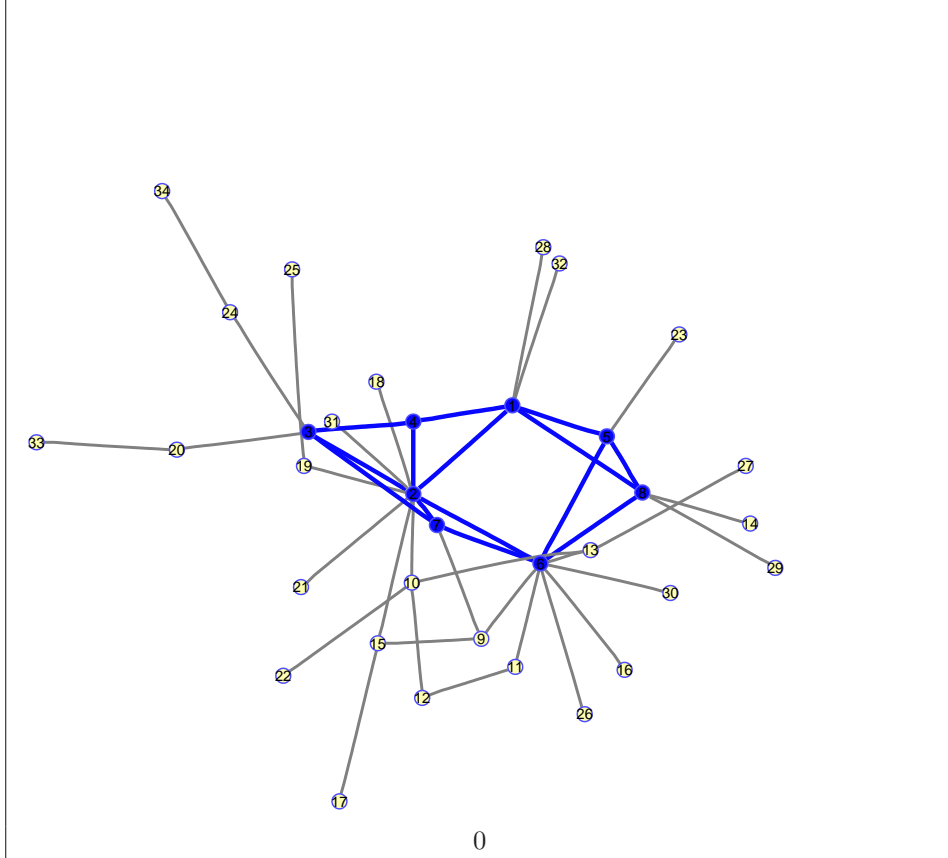


1

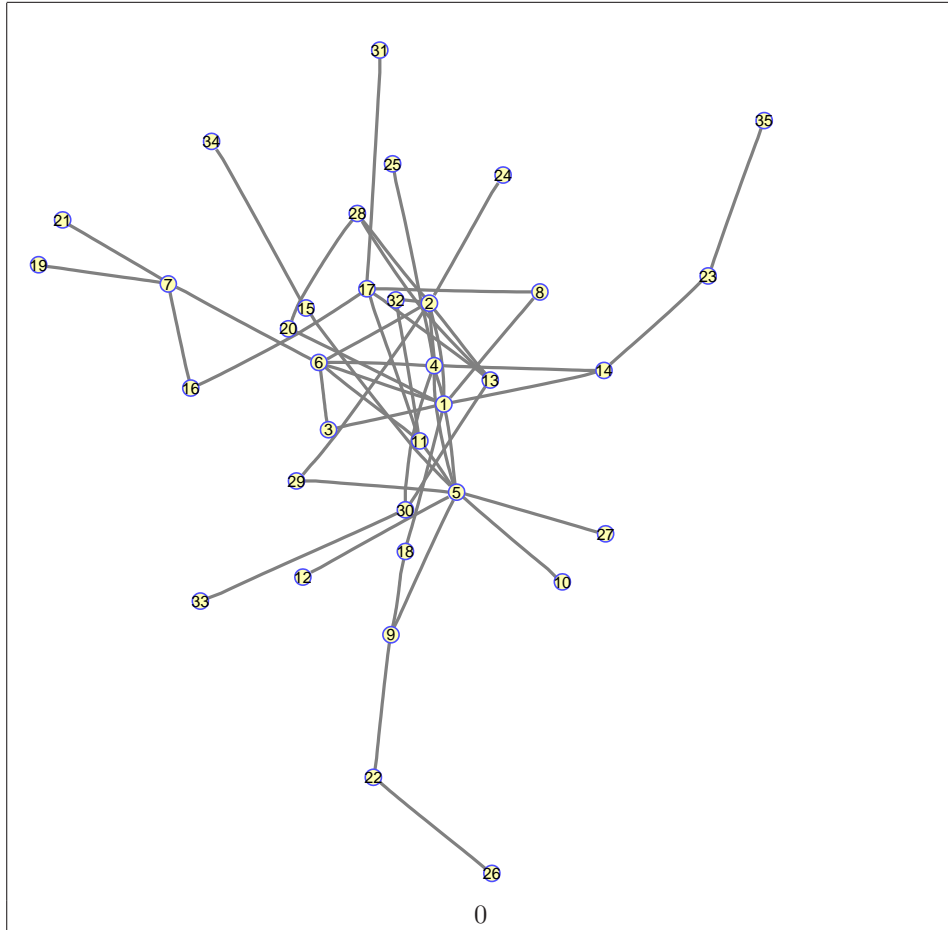


2

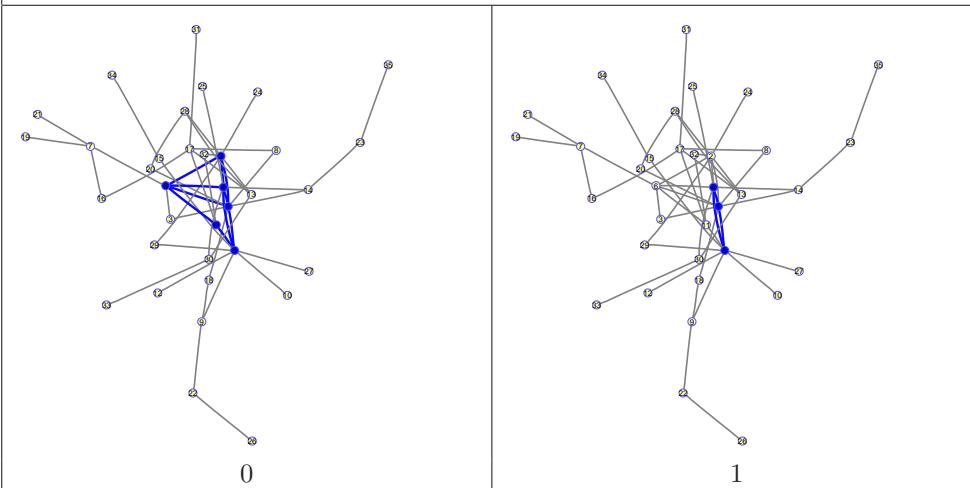
3-Assemblies



B.2.45 scaleFreeGraph(60,0.5,0.5,0.5,0.5,[1],[1],undirected)-3

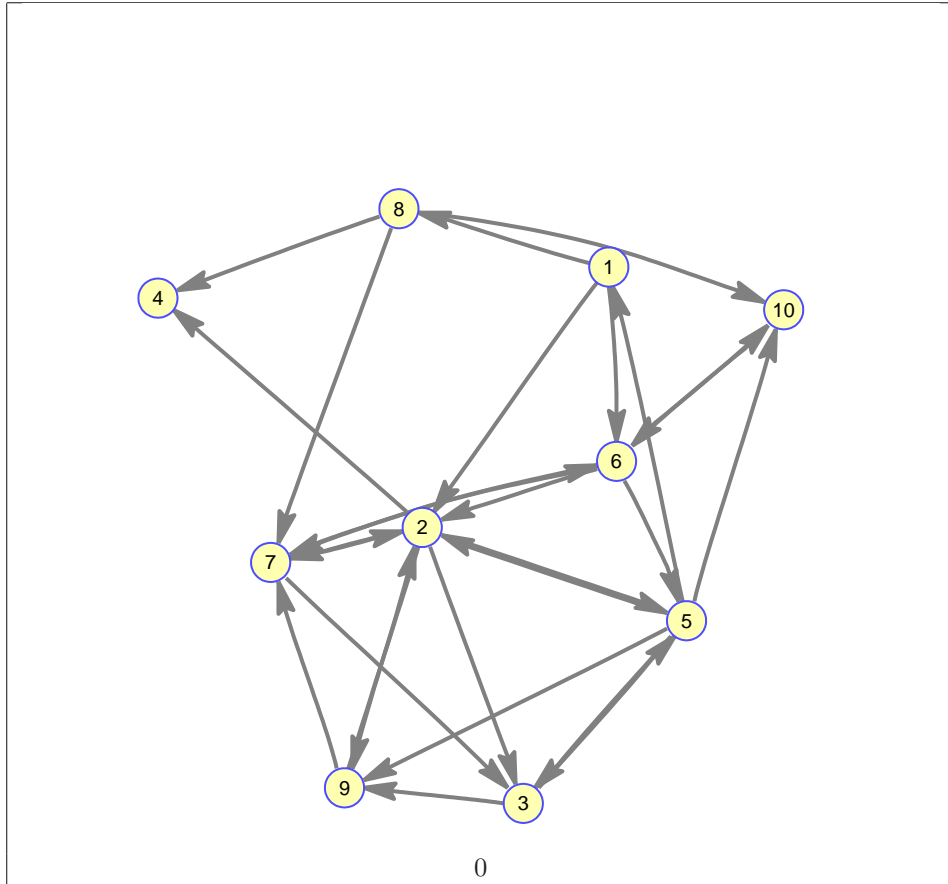


3-Assemblies



B.3 Directed Graphs

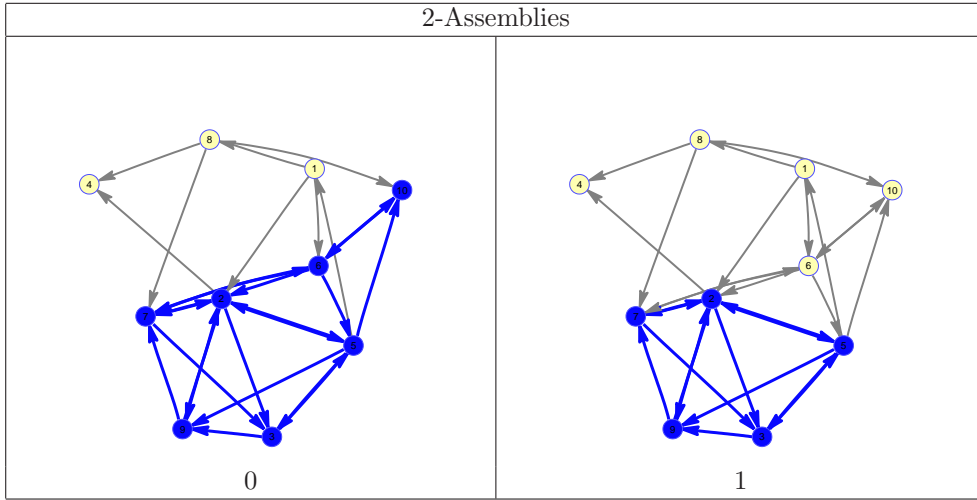
B.3.1 dir1



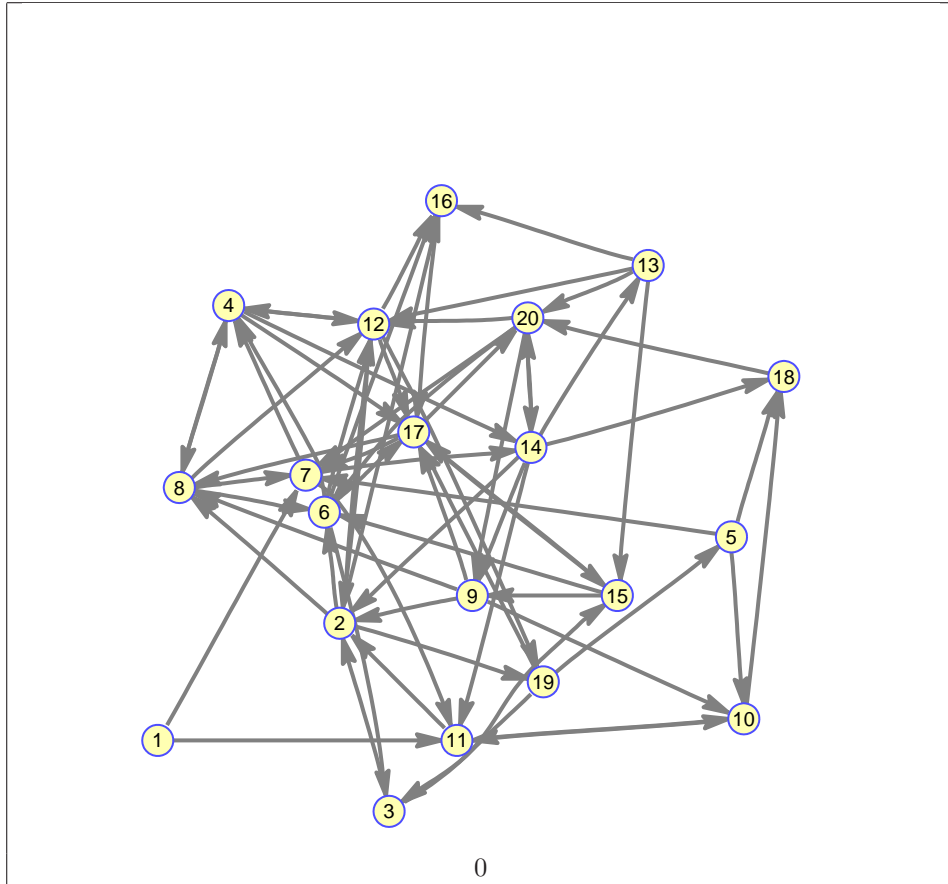
0	1	0	0	0	1	0	1	0	0
0	0	1	1	1	0	1	0	1	0
0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	1	1
0	1	0	0	1	0	1	0	0	1
0	1	1	0	0	1	0	0	0	0
0	0	0	1	0	0	1	0	0	1
0	1	0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0	0	0

(46)

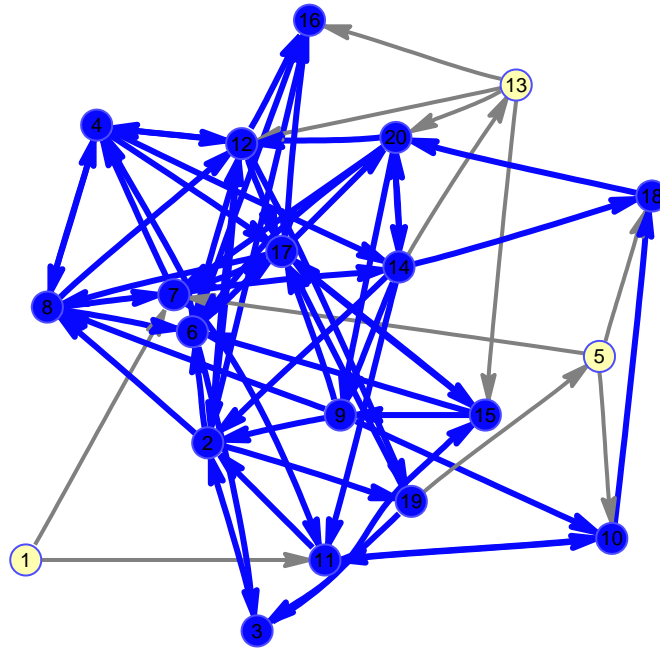
2-Assemblies



B.3.2 dir2

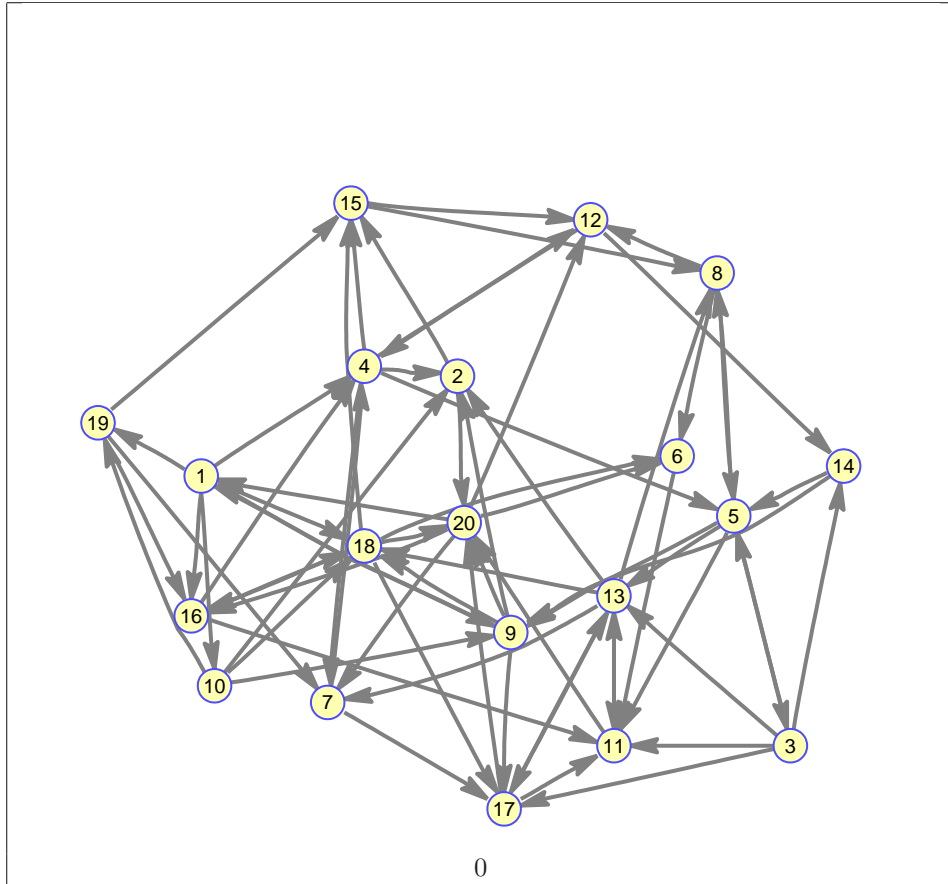


2-Assemblies

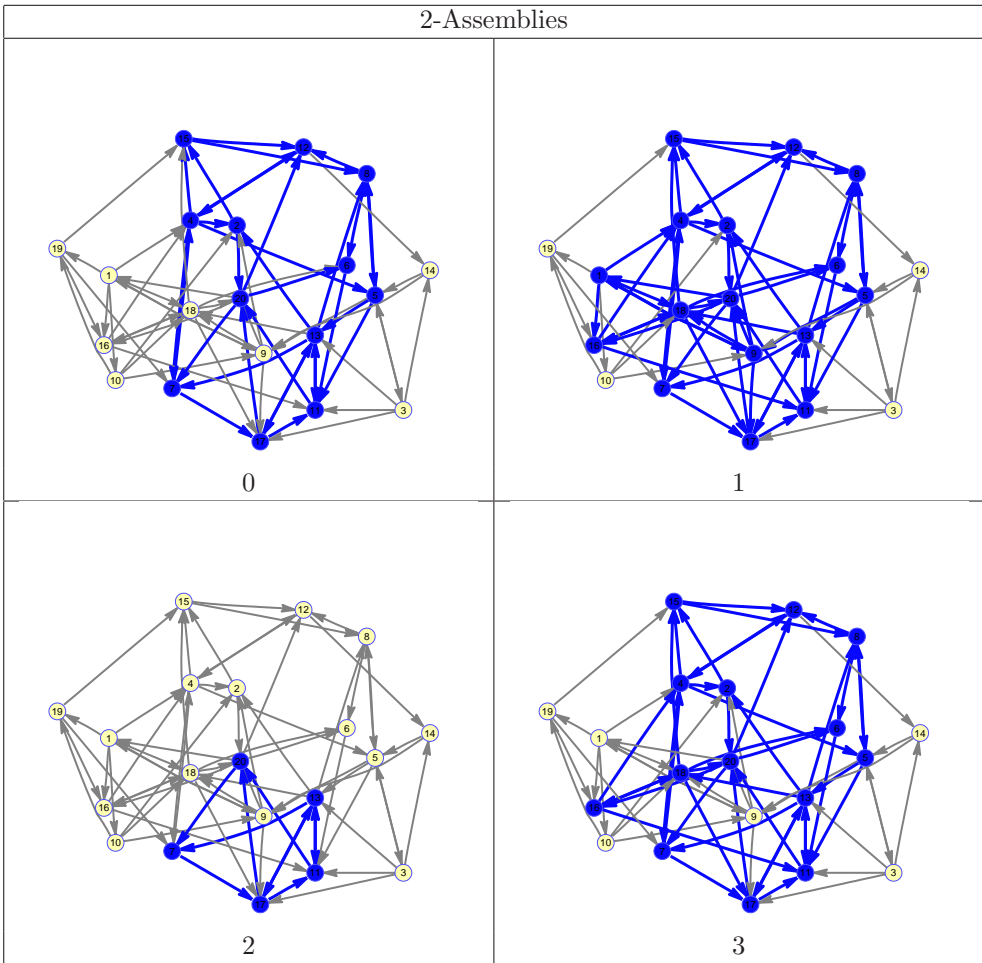


0

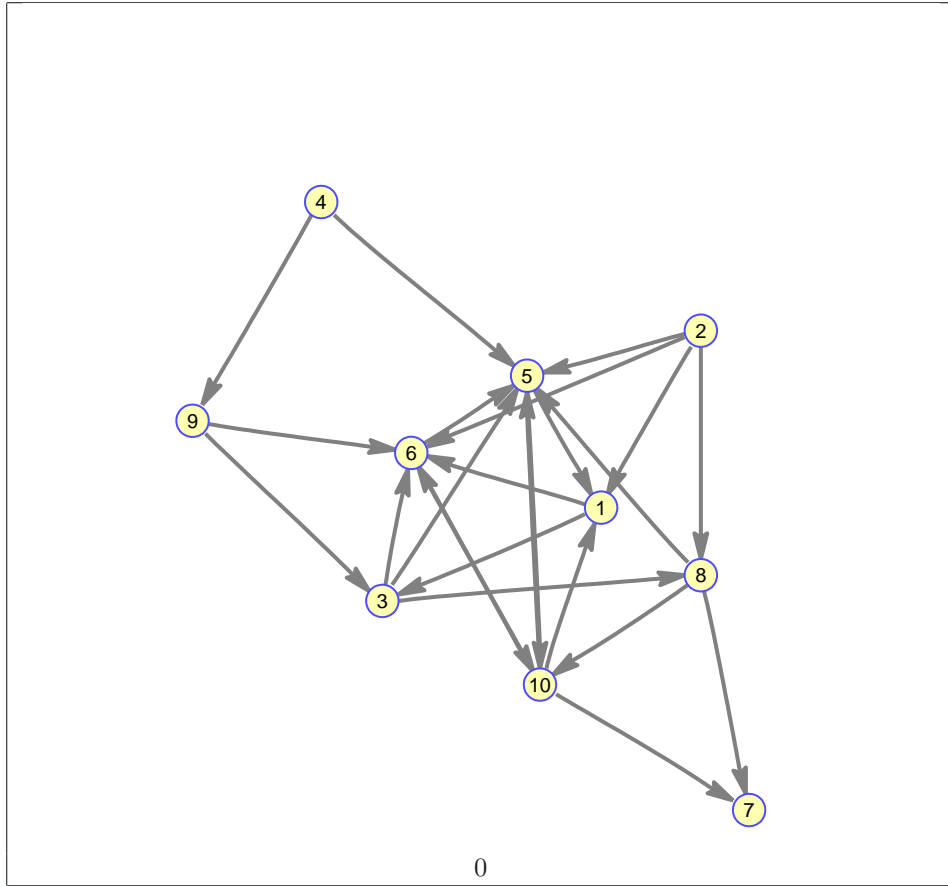
B.3.3 dir3



2-Assemblies



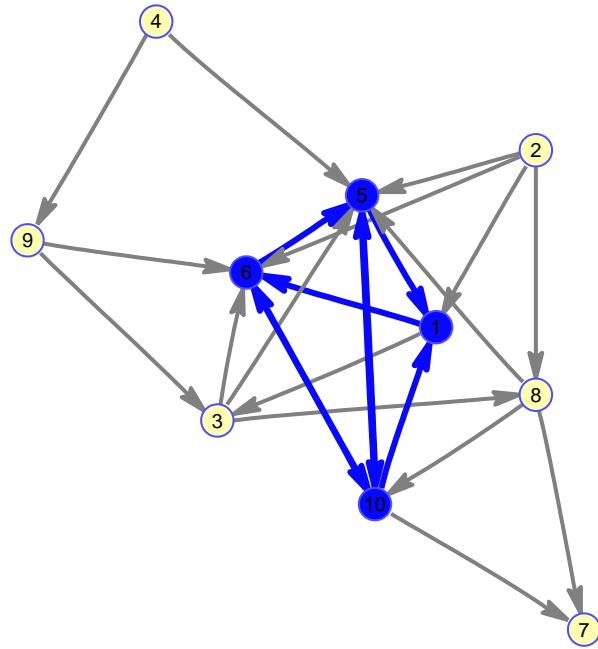
B.3.4 dir4



0	0	1	0	0	1	0	0	0	0
1	0	0	0	1	1	0	1	0	0
0	0	0	0	1	1	0	1	0	0
0	0	0	0	1	0	0	0	1	0
1	0	0	0	0	0	0	0	0	1
0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	0	0	1
0	0	1	0	0	1	0	0	0	0
1	0	0	0	1	1	1	0	0	0

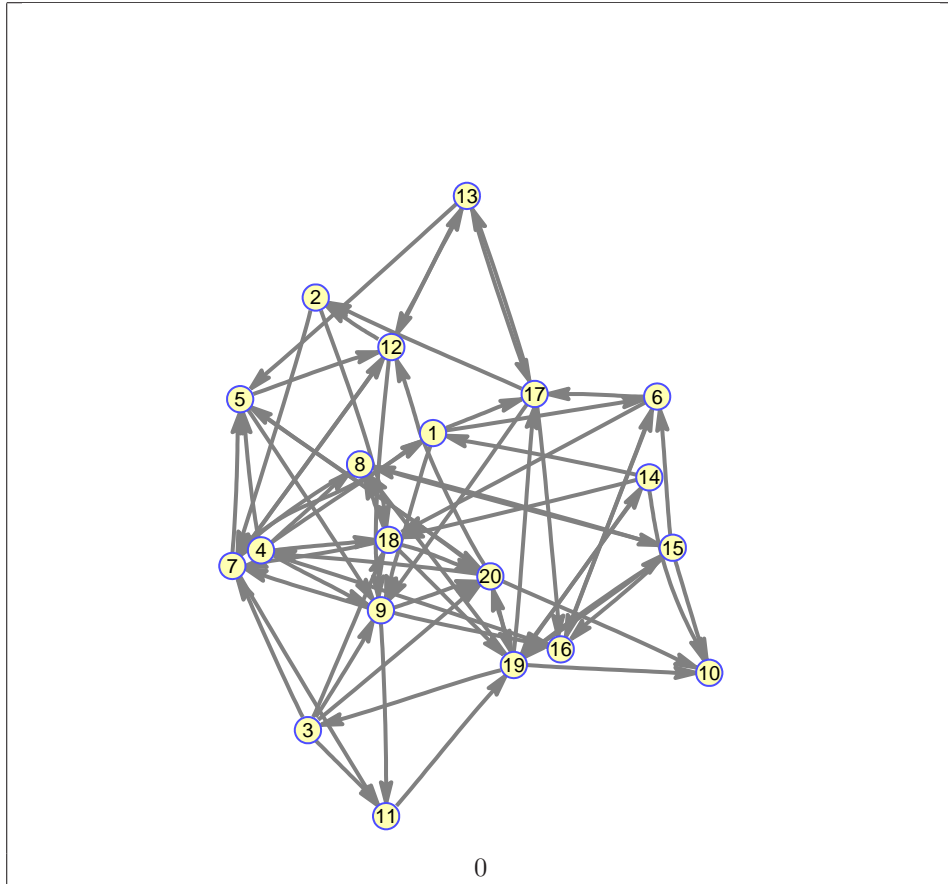
(49)

2-Assemblies



0

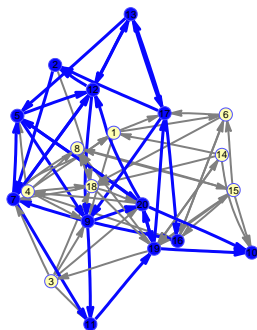
B.3.5 dir5



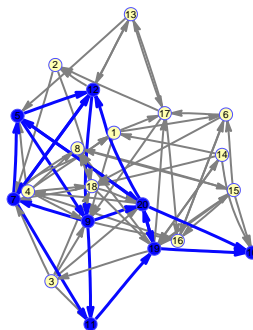
0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	1	0	1
1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	1	0	1	0	0
0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	1	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	1	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	1	0	0	0	0	1	0	1	0	0	0	1	1	0	1	0	0	1
0	0	0	1	1	0	0	0	0	1	0	1	0	0	0	0	0	1	0	0

(50)

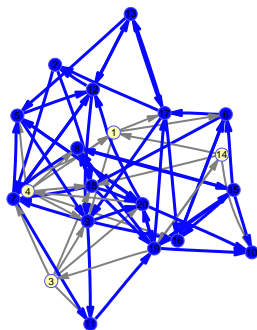
2-Assemblies



0

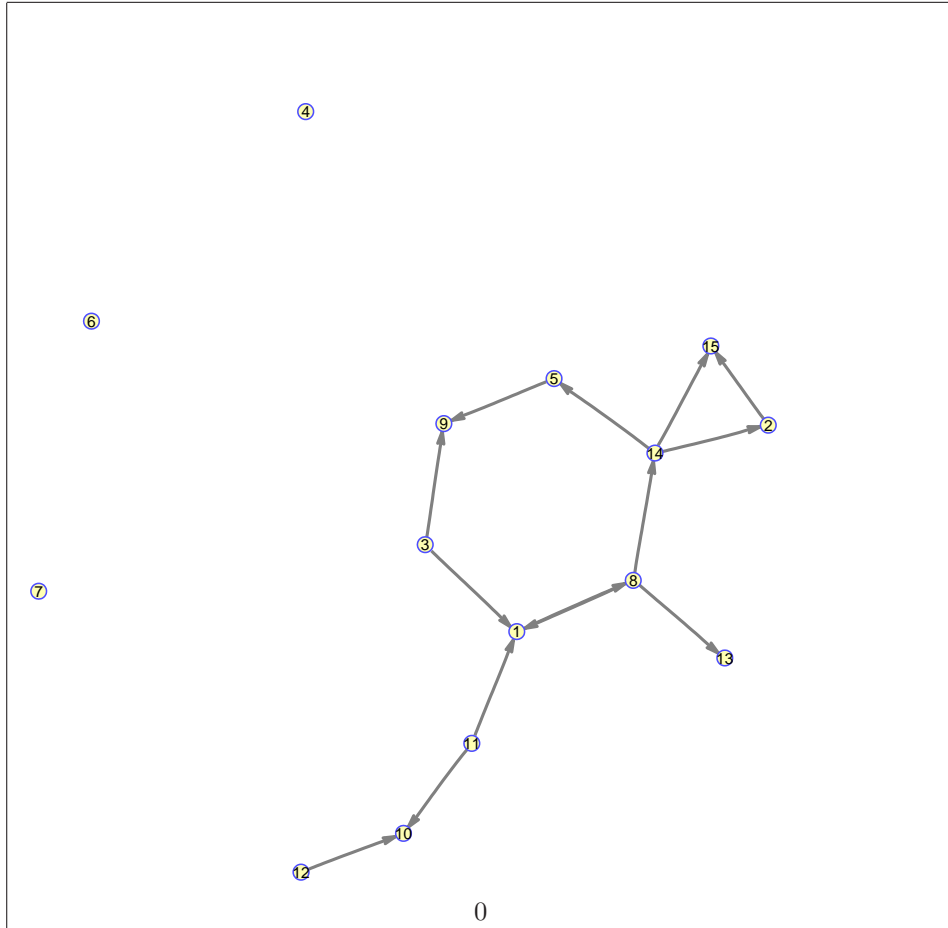


1



2

B.3.6 randomGraph(15,0.075,directed)-1



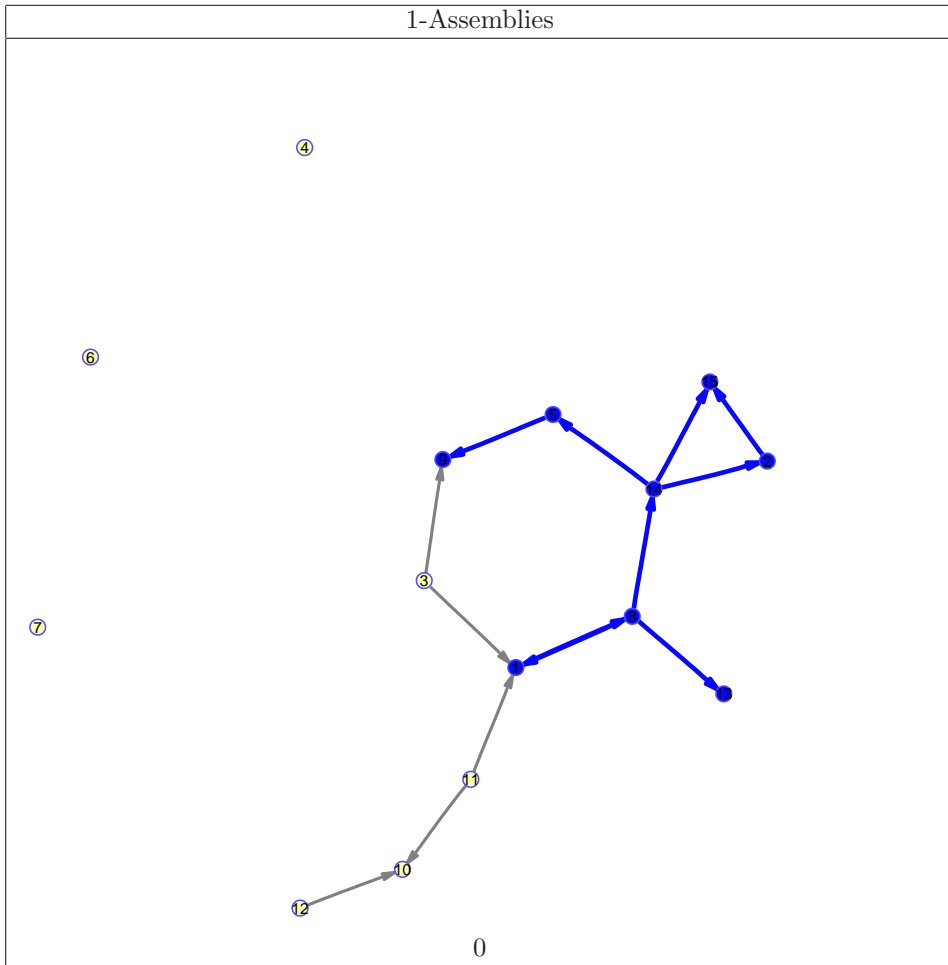
```

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
1 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 1 0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

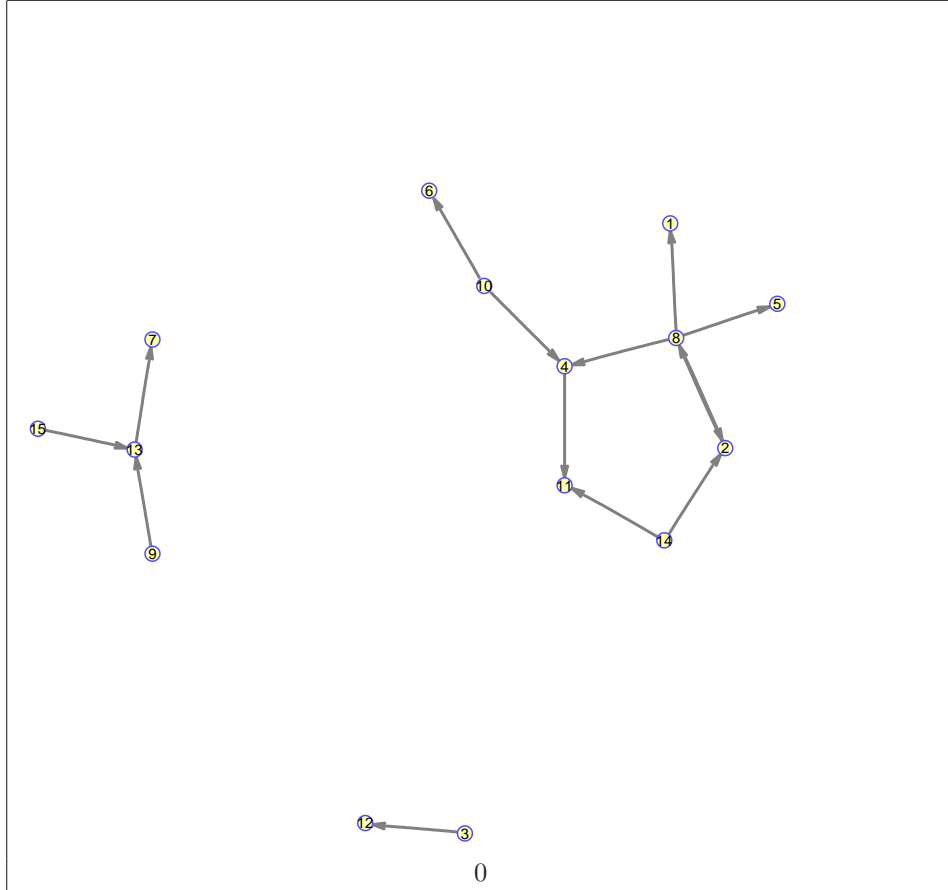
```

(51)

1-Assemblies



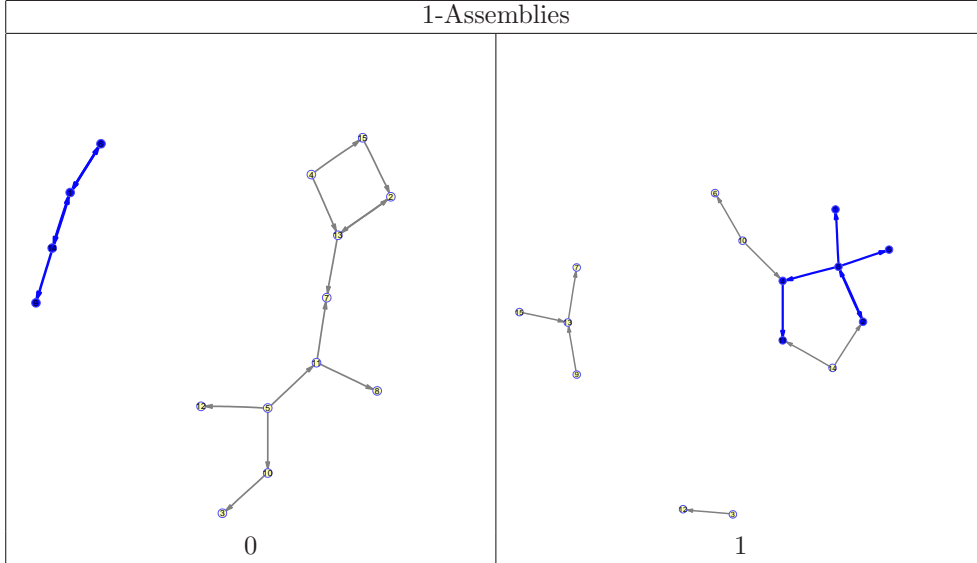
B.3.7 randomGraph(15,0.075,directed)-2



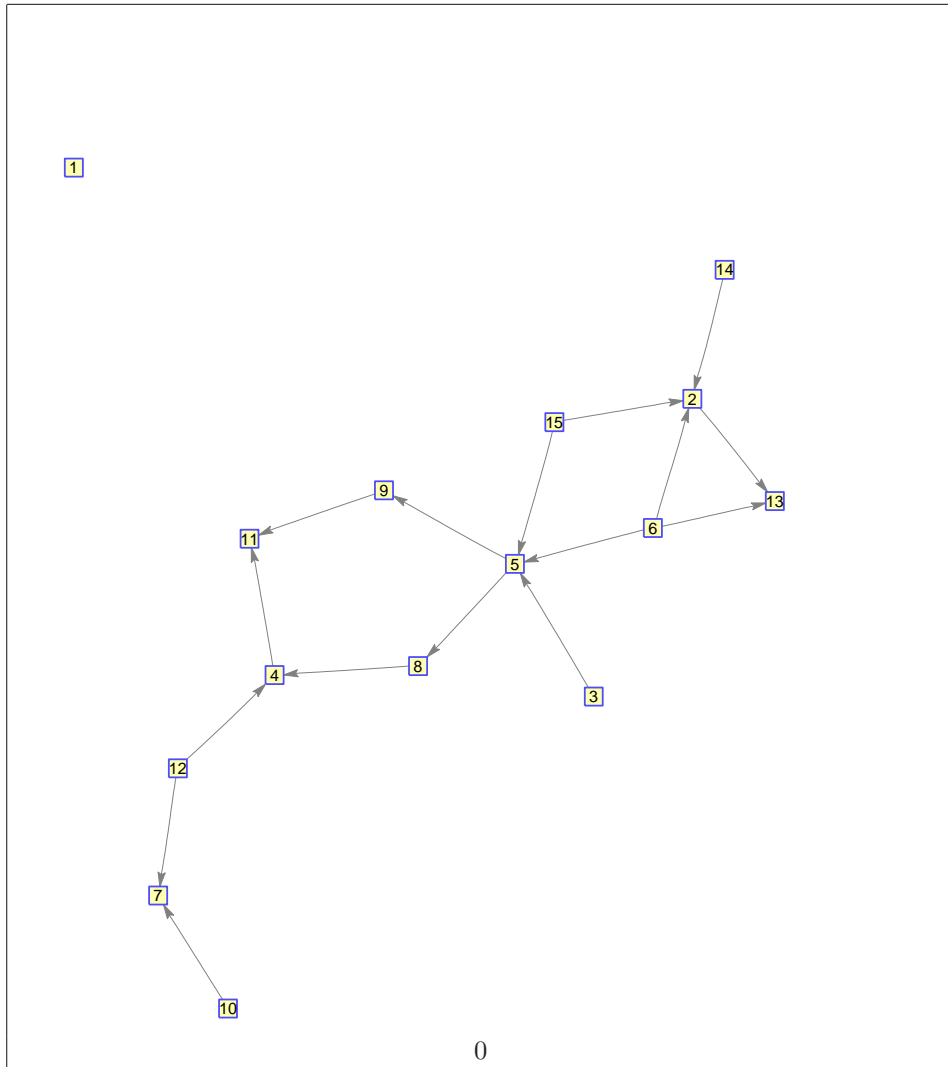
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

(52)

1-Assemblies



B.3.8 randomGraph(15,0.075,directed)-3



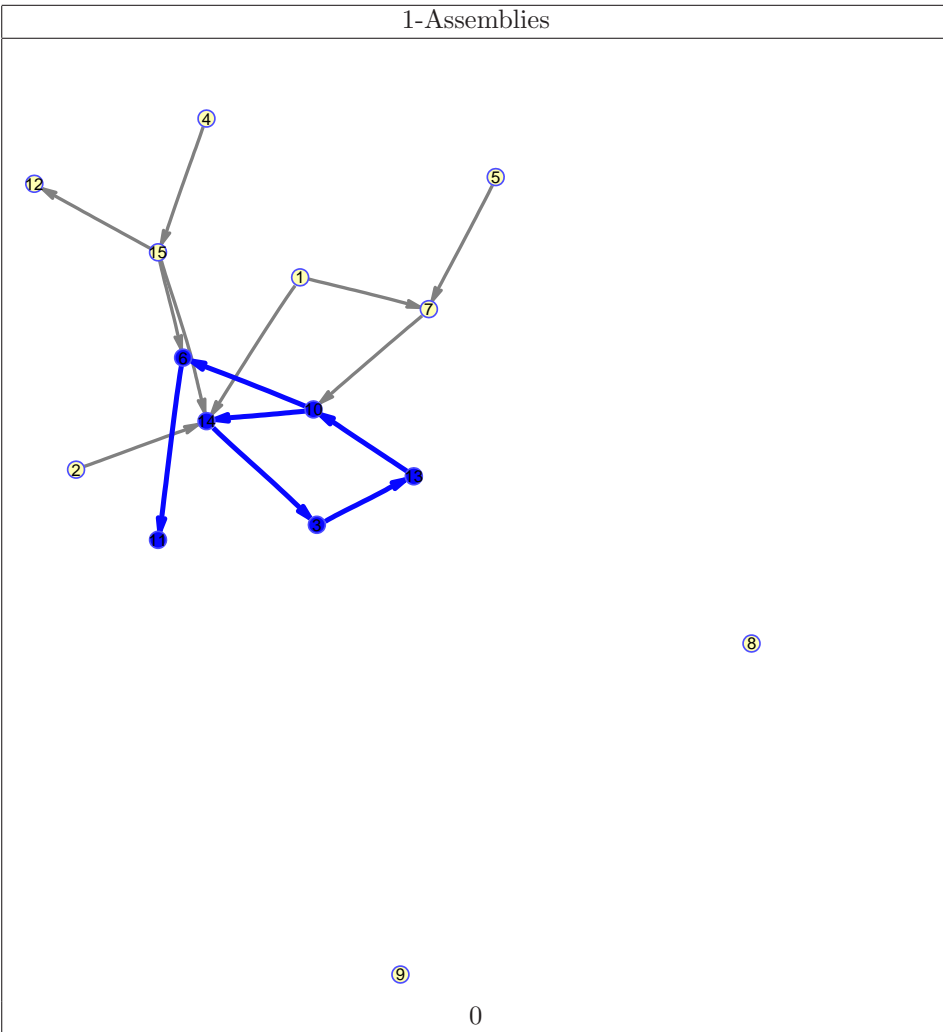

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
0 1 0 0 1 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 1 0 0 0 0 0 0 0 0 0 0

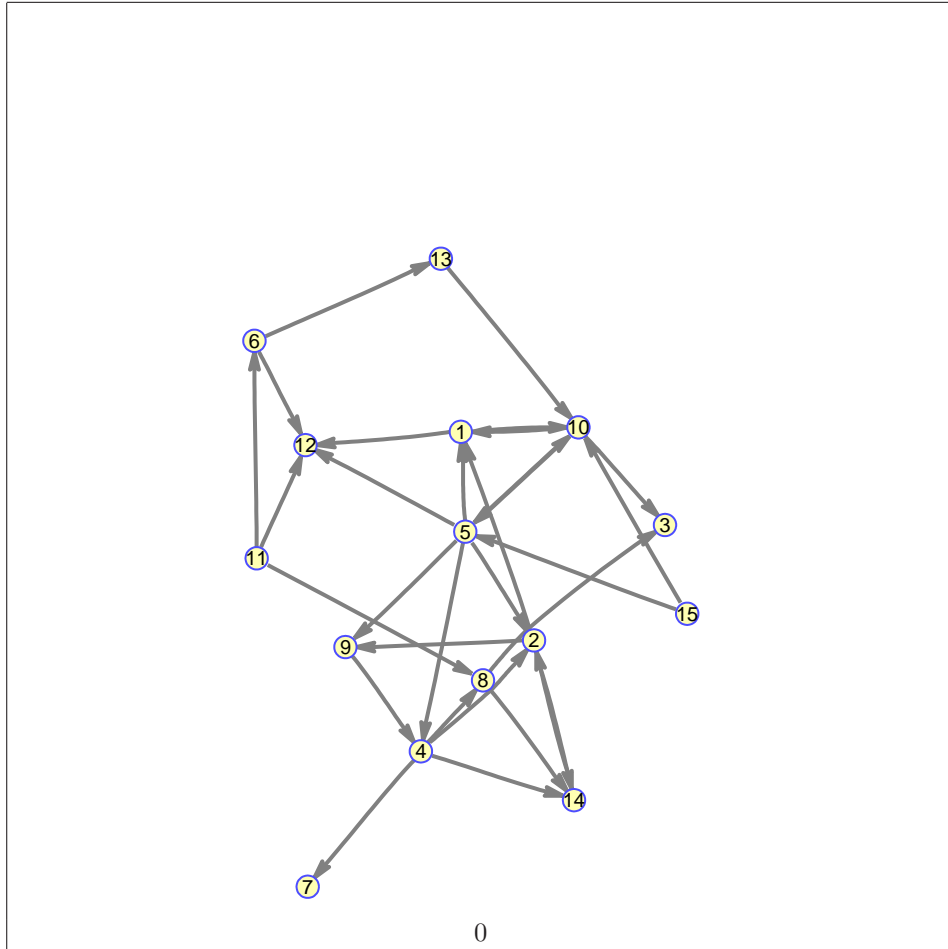
```

(53)

1-Assemblies



B.3.9 randomGraph(15,0.15,directed)-1



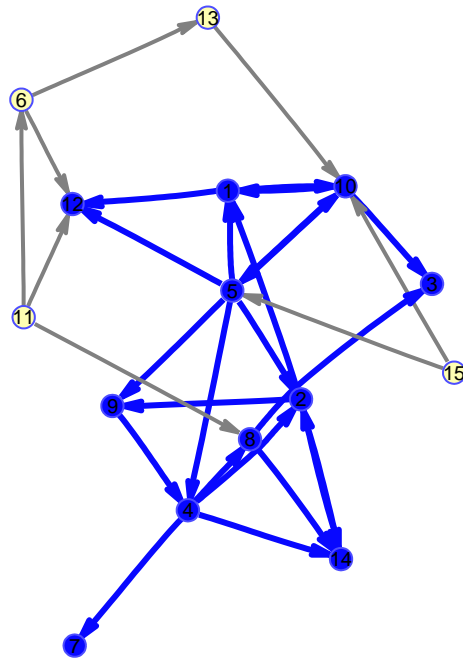
```

0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
1 0 0 0 0 0 0 0 1 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 1 1 0 0 0 0 0 1 0
1 1 0 1 0 0 0 0 1 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 1 0 0 0 0 0

```

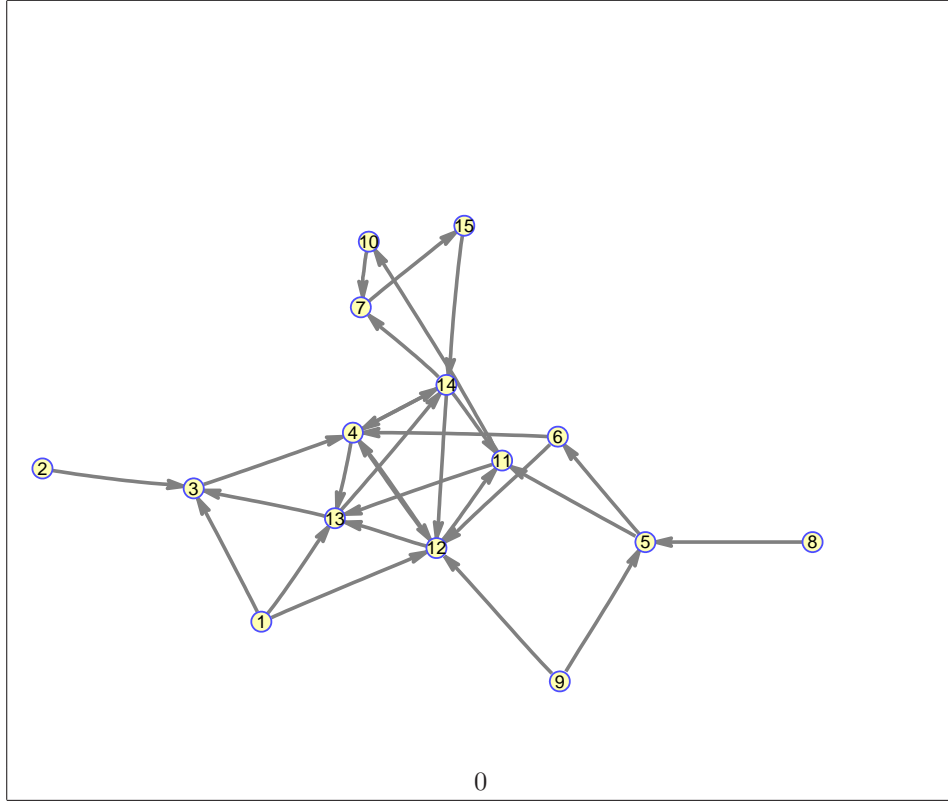
(54)

1-Assemblies



0

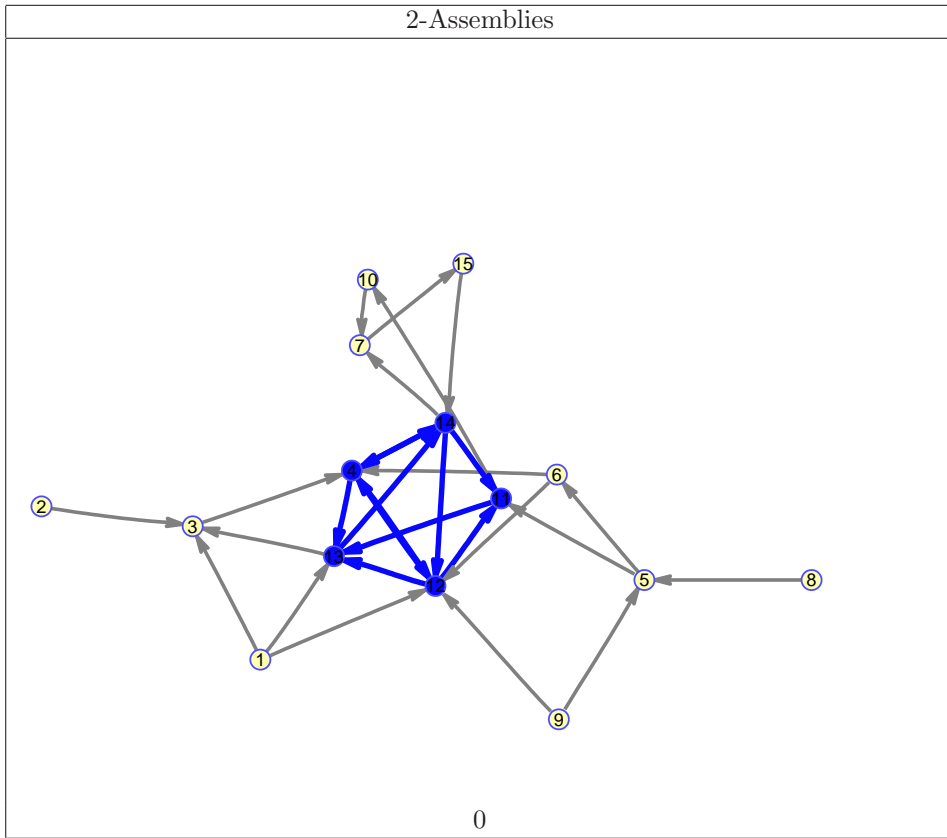
B.3.10 randomGraph(15,0.15,directed)-2



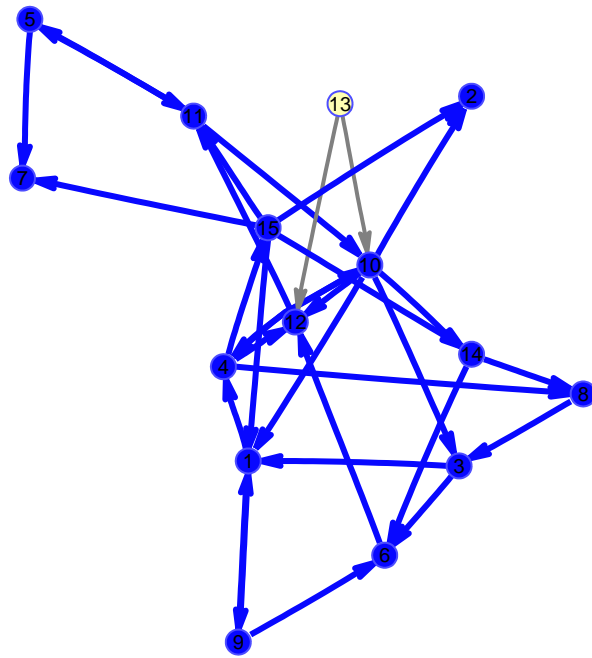
0	0	1	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	1	0	0	0	0	0	0	1	0	1	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	1	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

(55)

2-Assemblies

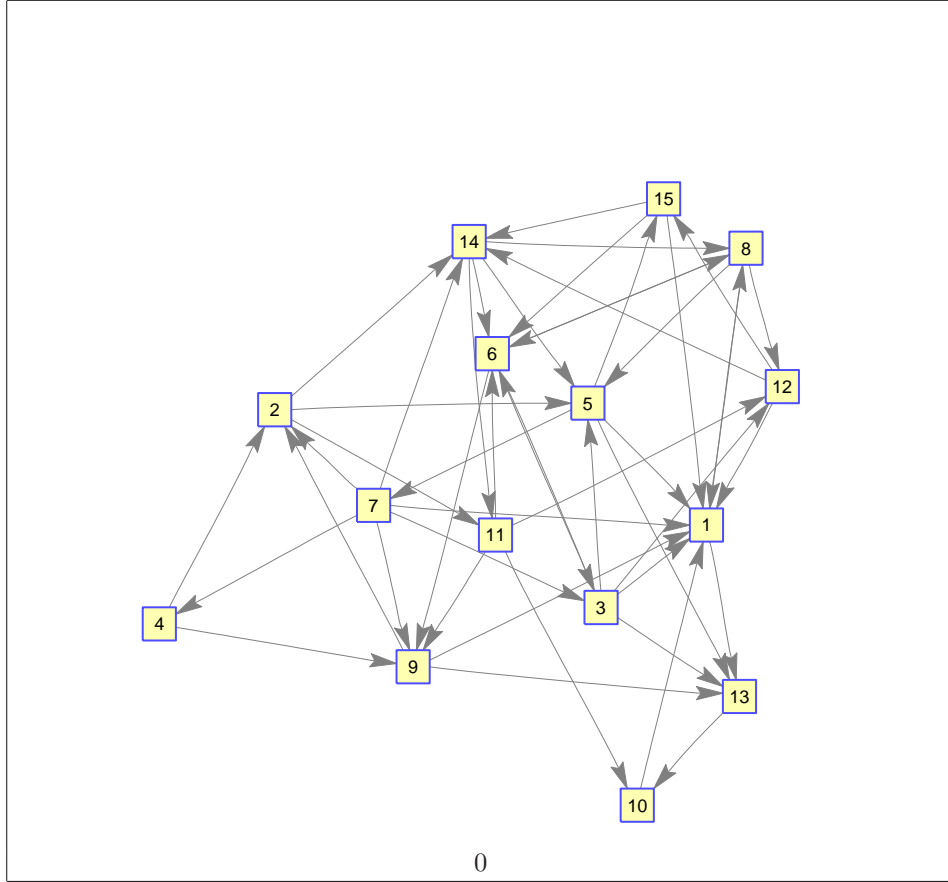


1-Assemblies



0

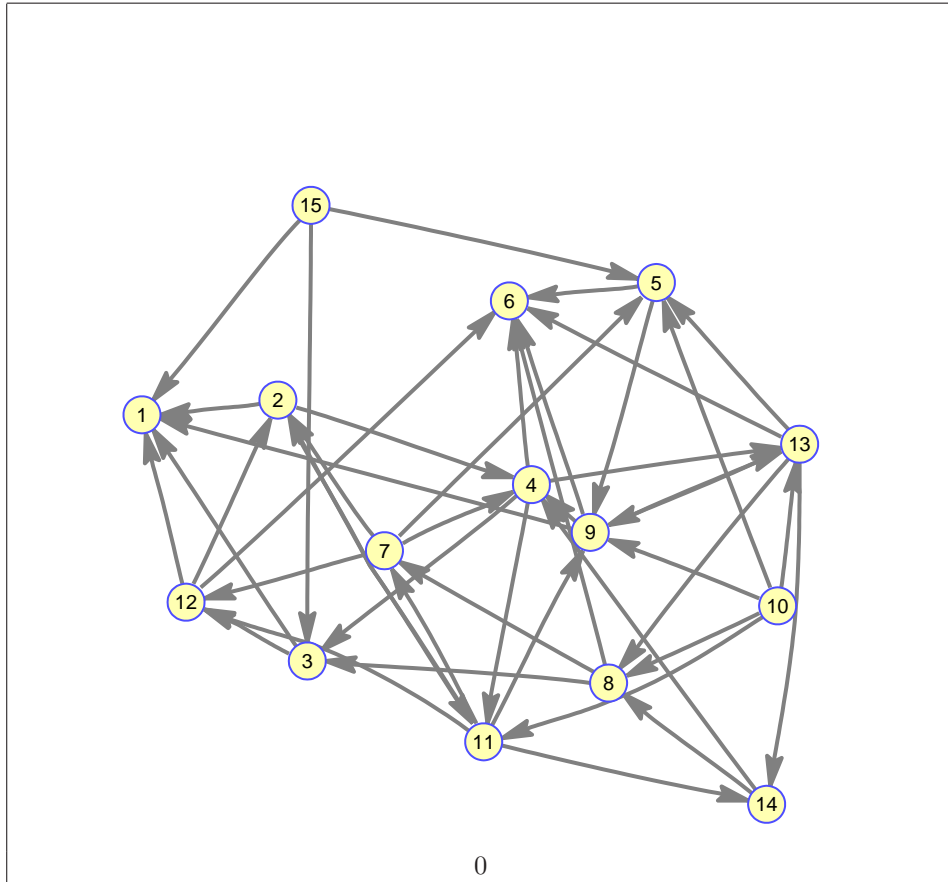
B.3.12 randomGraph(15,0.25,directed)-1



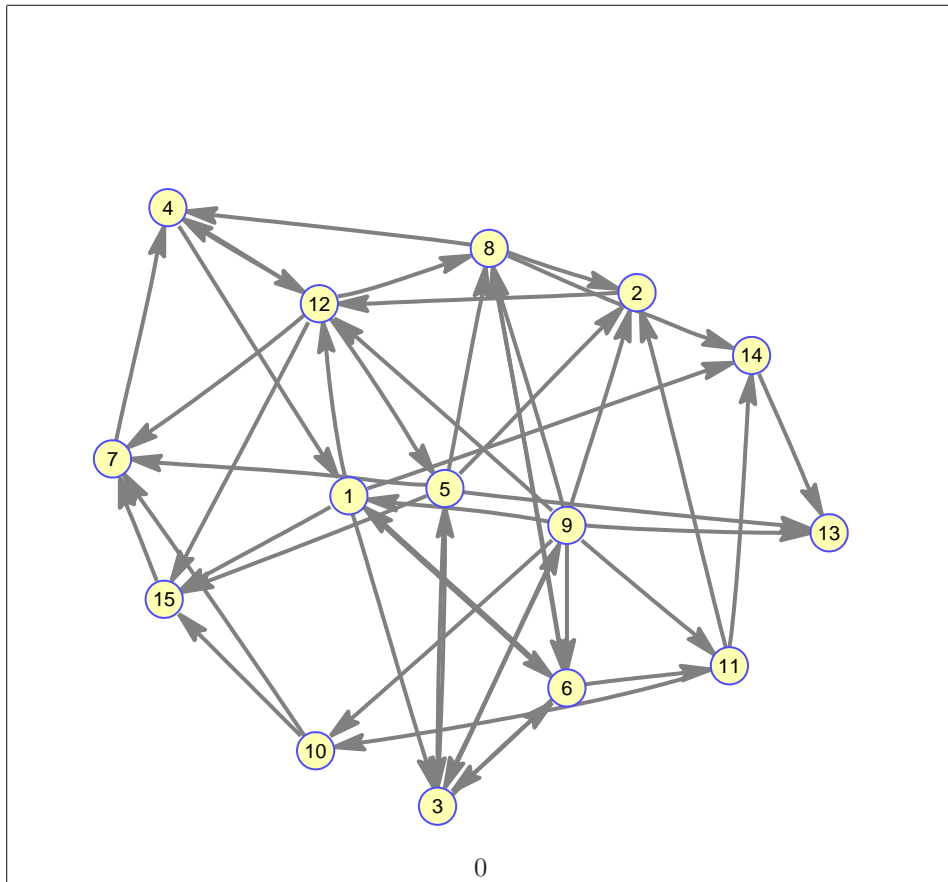
0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	
0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0
1	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
1	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	1	0	0	0	0	0	1	0
1	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	1	1	0	1	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	1	0	1	0	0	1	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0

(57)

B.3.13 randomGraph(15,0.25,directed)-2



B.3.14 randomGraph(15,0.25,directed)-3

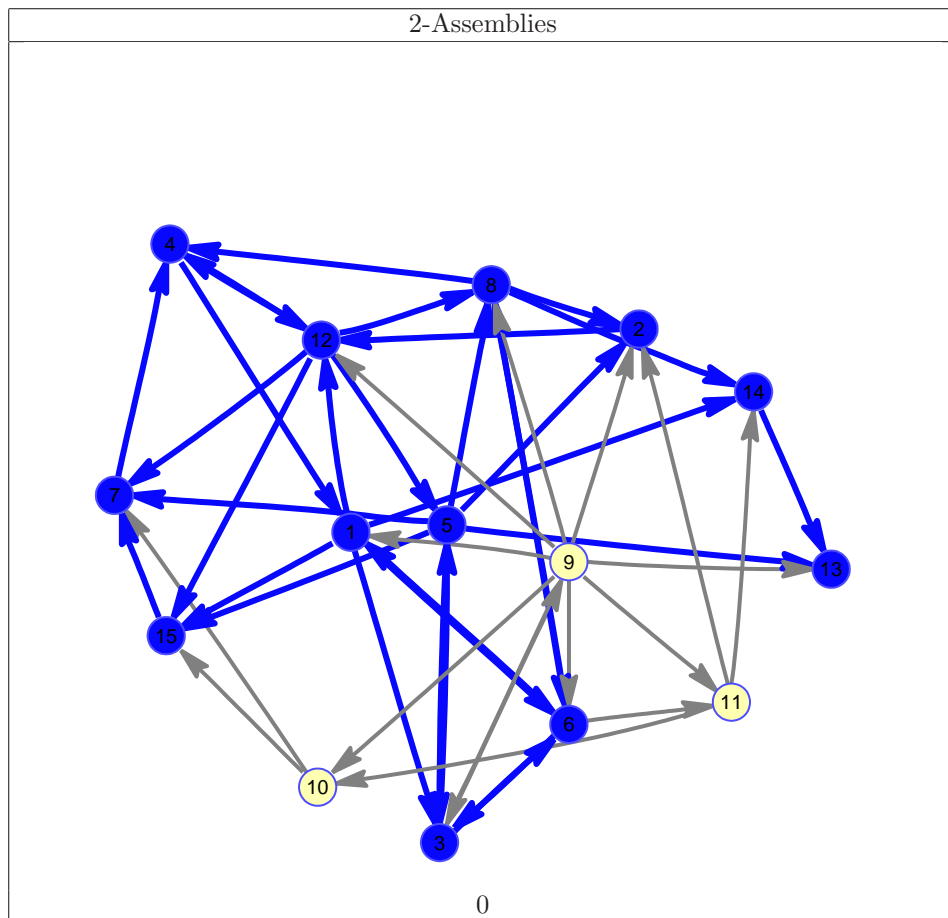


```

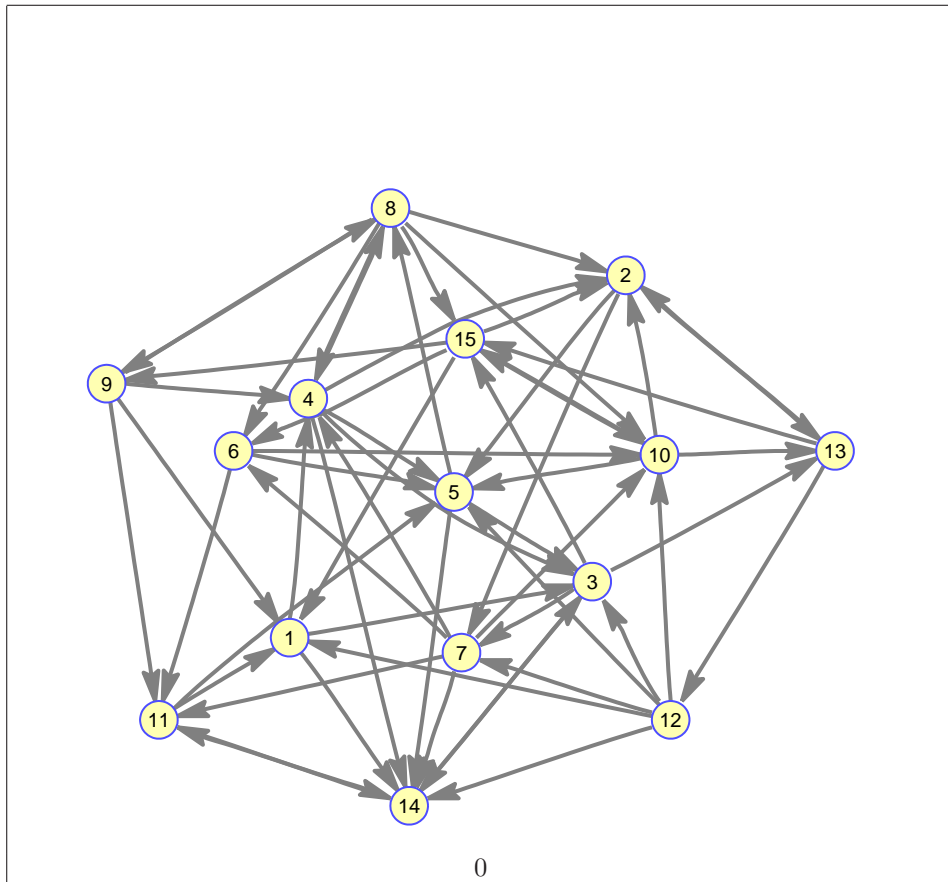
0 0 1 0 0 1 0 0 0 0 0 1 0 1 1
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 1 1 0 0 1 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 1 1 0 0 0 1 1 0 0 0 0 1 0 1
1 0 1 0 0 0 0 1 0 0 1 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 1 0 1 0 1 0 0 0 0 0 0 0 1 0
1 1 1 0 0 1 0 1 0 1 1 1 1 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 1
0 1 0 0 0 0 0 0 0 1 0 0 0 1 0
0 0 0 1 1 0 1 1 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0

```

(59)



B.3.15 randomGraph(15,0.325,directed)-1

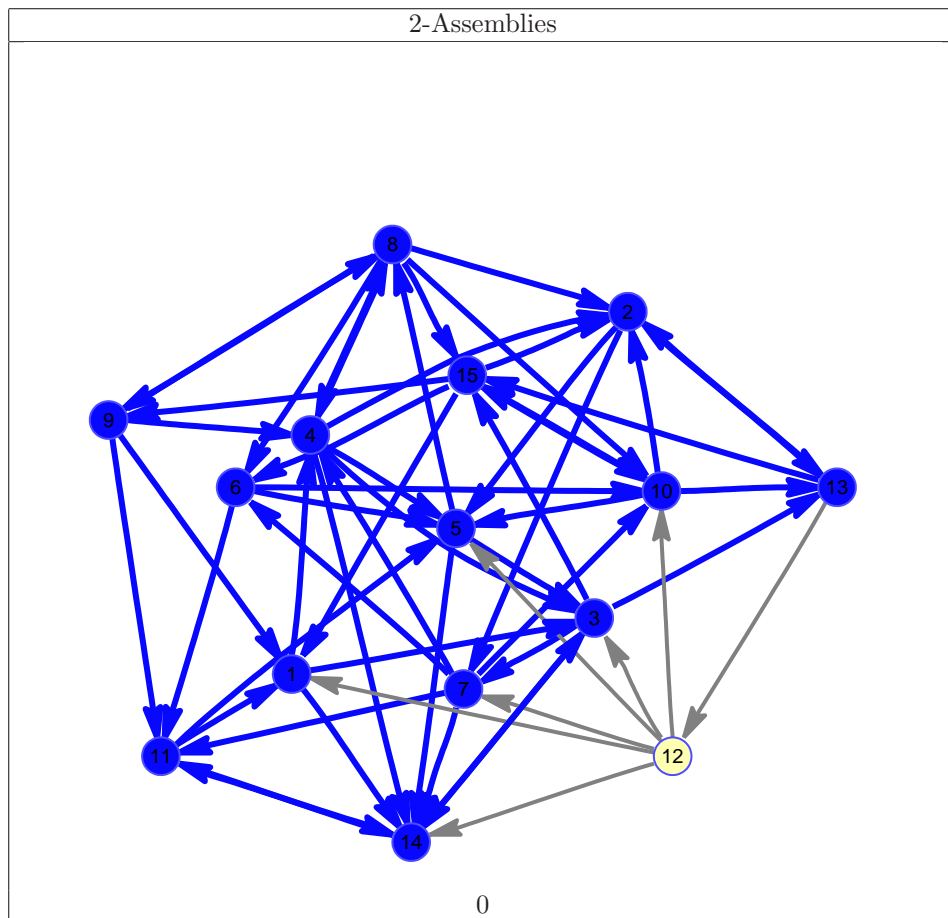



```

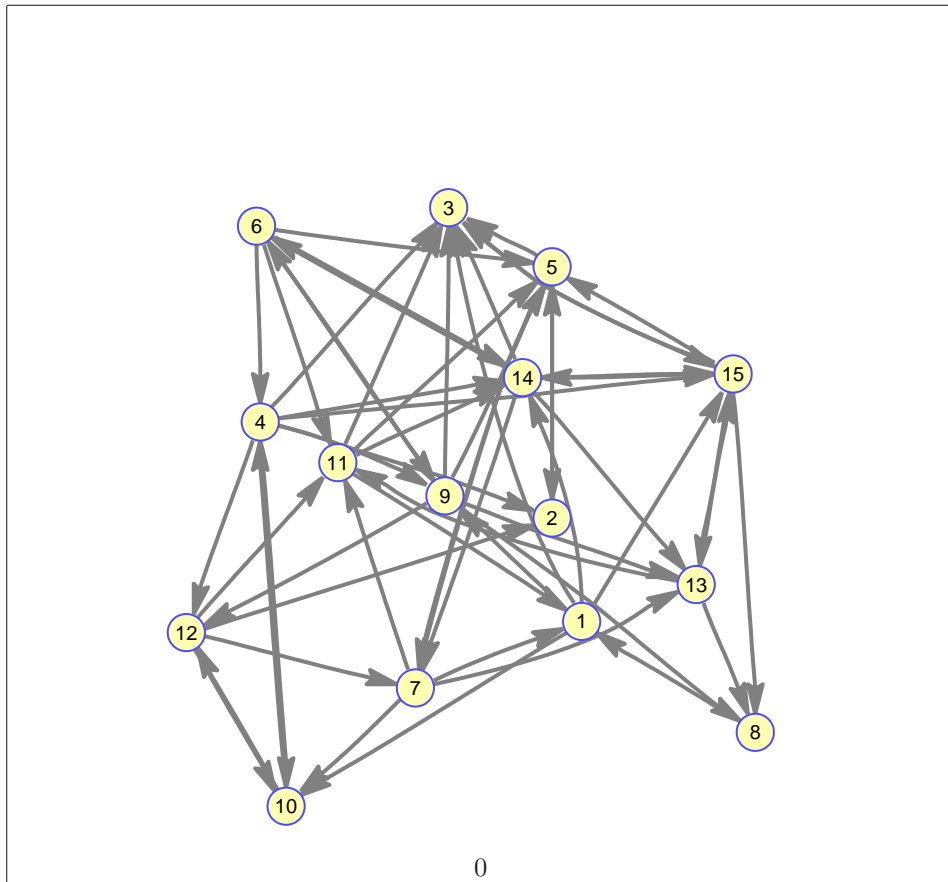
0 0 1 1 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 1 0 1 0 0 0 0 0 1 0 0
0 0 0 0 0 0 1 0 0 0 0 0 1 1 1
0 1 1 0 1 0 0 1 0 0 0 0 0 1 0
0 0 1 0 0 0 0 1 0 0 0 0 0 1 0
0 0 0 0 1 0 0 0 0 1 1 0 0 0 0
0 0 0 1 0 1 0 0 0 1 1 0 0 1 0
0 1 0 1 0 1 0 0 1 1 0 0 0 0 1
1 0 0 1 0 0 0 1 0 0 1 0 0 0 0
0 1 0 0 1 0 0 0 0 0 0 0 1 0 1
1 0 0 0 1 0 0 0 0 0 0 0 0 1 0
1 0 1 0 1 0 1 0 0 1 0 0 0 1 0
0 1 0 0 0 0 0 0 0 0 0 1 0 0 1
0 0 1 0 0 0 0 0 0 0 1 0 0 0 0
1 1 0 0 0 1 0 0 1 1 0 0 0 0 0

```

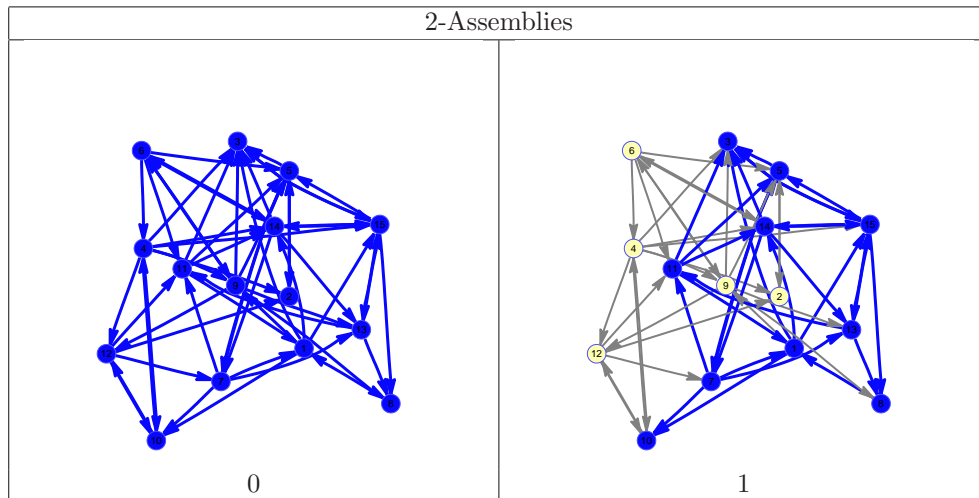
(60)



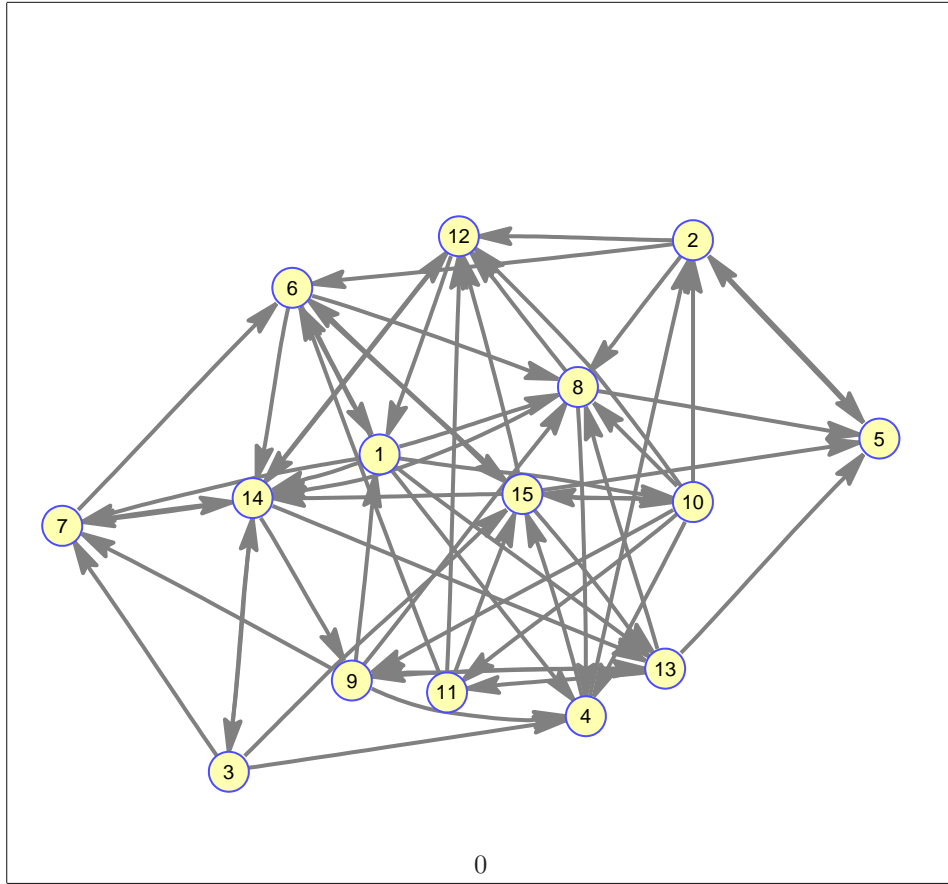
B.3.16 randomGraph(15,0.325,directed)-2



$$\begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{pmatrix} \tag{61}$$



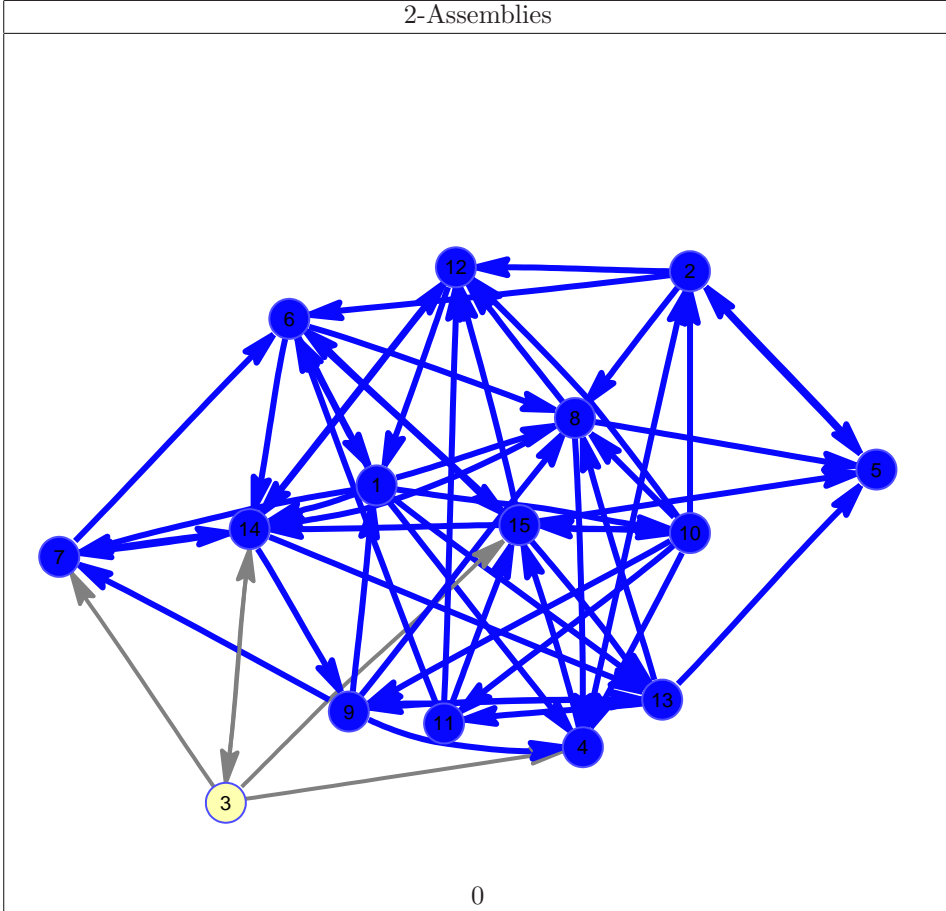
B.3.17 randomGraph(15,0.325,directed)-3



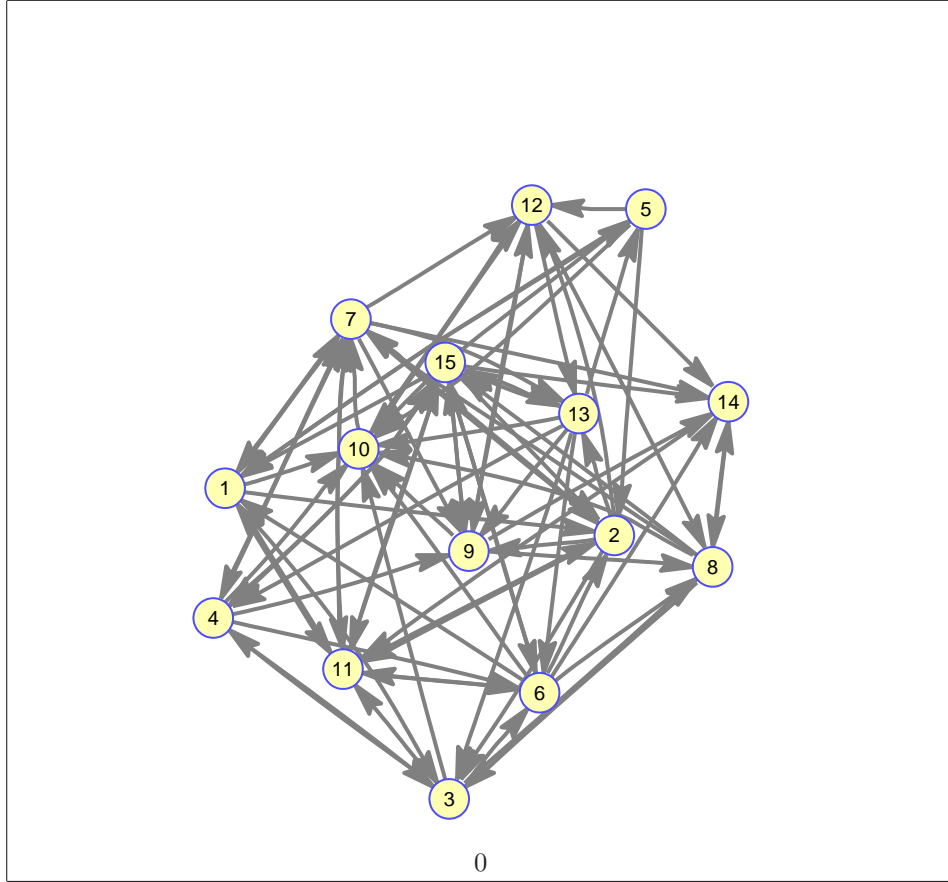
0	0	0	1	0	1	1	1	0	1	0	0	1	1	0
0	0	0	0	1	1	0	1	0	0	0	1	0	0	0
0	0	0	1	0	0	1	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	1	1	0	0	0	0	0	0	1	0	1	0
1	0	0	1	0	0	1	1	0	0	0	0	1	0	0
0	1	0	1	0	0	0	1	1	0	1	1	0	0	1
0	0	0	0	0	1	0	0	0	0	0	1	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	1	0	0	1	1	0	1	0	0	0	0
0	0	1	0	0	0	1	0	1	0	0	1	1	0	0
0	0	0	0	1	1	0	0	0	1	0	1	1	1	0

(62)

2-Assemblies



B.3.18 randomGraph(15,0.5,directed)-1



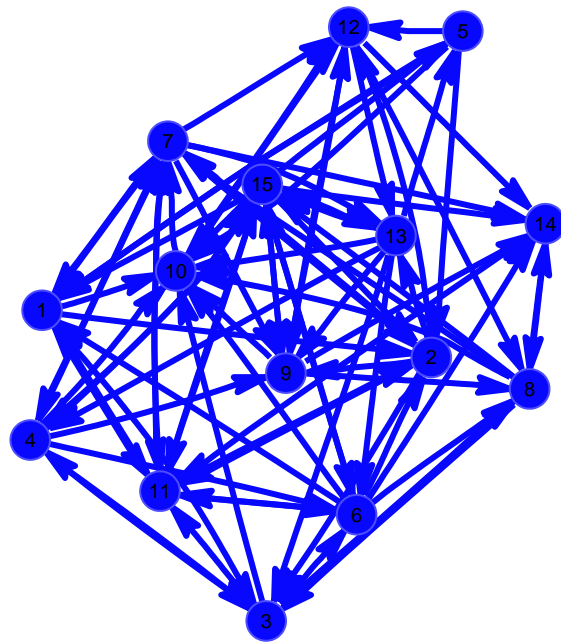
```

0 1 1 0 1 0 1 0 0 1 1 0 0 0 0
0 0 1 0 0 0 1 0 1 0 1 1 1 0 0
0 0 0 1 0 1 0 1 0 1 1 0 0 0 0
0 0 1 0 0 1 1 0 1 1 0 0 0 0 1
1 1 0 0 0 0 0 0 0 1 0 1 0 0 0
1 1 0 0 0 0 0 1 0 1 1 0 0 1 1
1 1 0 1 0 0 0 0 1 0 1 1 1 1 0
0 0 1 0 0 0 1 0 0 1 0 0 0 1 1
0 0 0 0 0 0 0 1 0 1 0 1 0 1 1
0 0 0 0 0 0 1 0 0 0 0 1 0 0 0
1 1 0 0 0 1 1 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 1 1 1 0 0 1 1 0
0 0 1 1 1 1 0 0 1 1 0 0 0 0 1
0 0 0 0 0 0 0 1 0 0 1 0 0 0 0
1 1 0 1 1 1 0 0 1 1 1 0 1 1 0

```

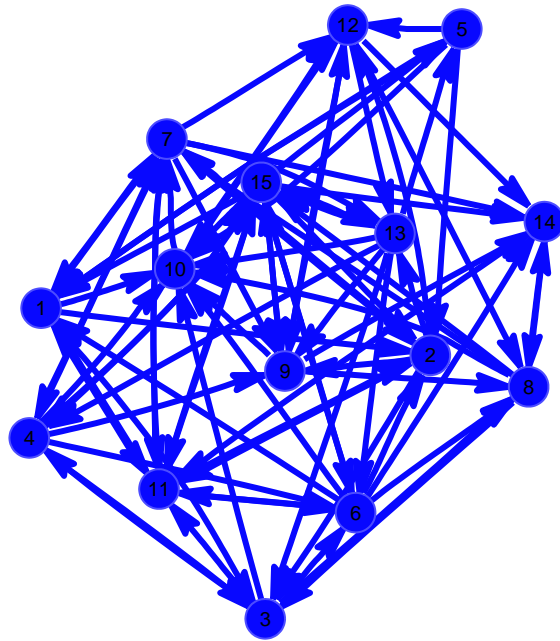
(63)

2-Assemblies



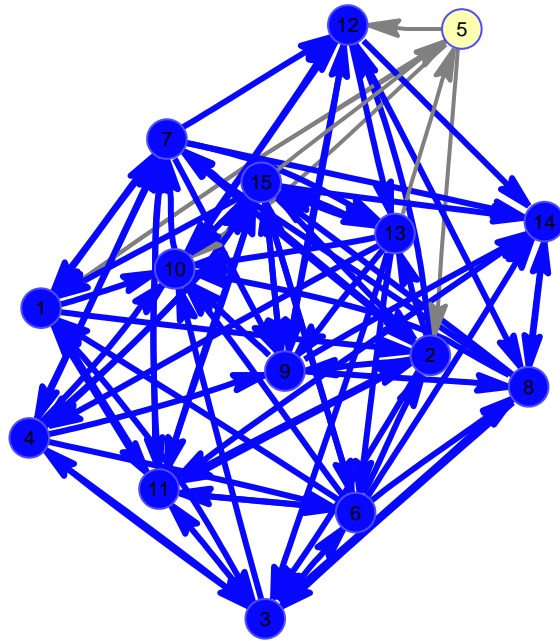
0

3-Assemblies



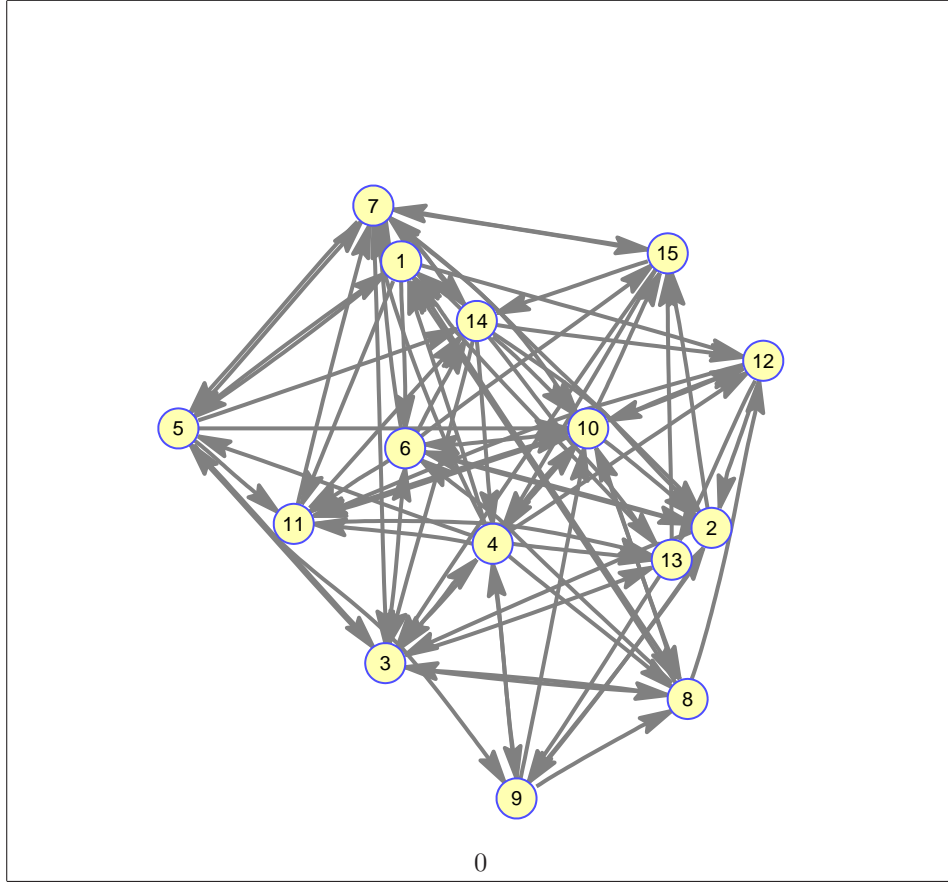
0

4-Assemblies



0

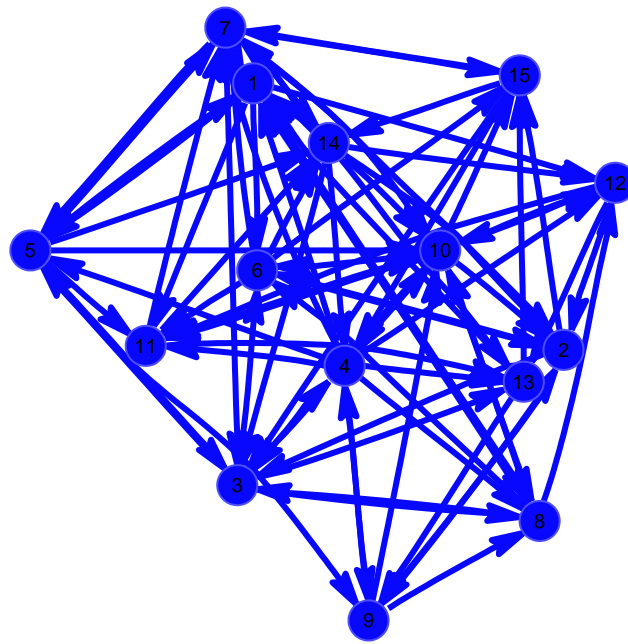
B.3.19 randomGraph(15,0.5,directed)-2



0	0	0	0	1	1	0	1	0	0	1	1	0	0	0
0	0	1	0	0	1	1	0	1	0	0	0	0	0	1
0	0	0	1	1	1	0	1	0	0	0	0	1	0	0
1	0	1	0	1	0	1	1	1	1	1	1	1	0	0
1	0	1	0	0	0	1	0	1	1	1	0	0	1	0
0	1	0	0	0	0	1	0	0	0	1	0	0	1	1
1	0	1	0	1	0	0	0	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	1	0	1	0	0	0	0	0
0	1	0	1	0	1	0	1	0	0	1	1	0	0	1
0	0	0	0	0	0	1	0	0	1	0	0	1	1	0
0	1	0	0	0	0	0	0	0	1	1	0	1	0	0
1	0	0	0	0	0	0	0	1	0	0	0	0	0	1
1	1	1	1	0	0	1	0	0	1	0	1	1	0	0
0	0	1	1	0	0	1	0	0	0	0	0	0	1	0

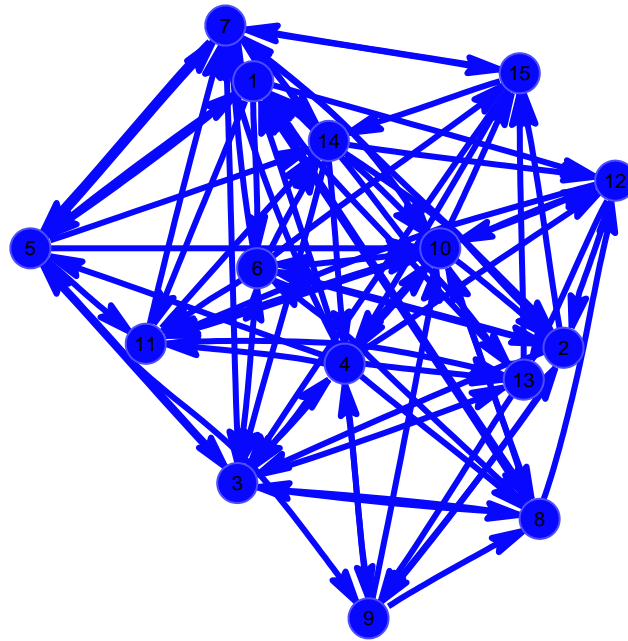
(64)

3-Assemblies



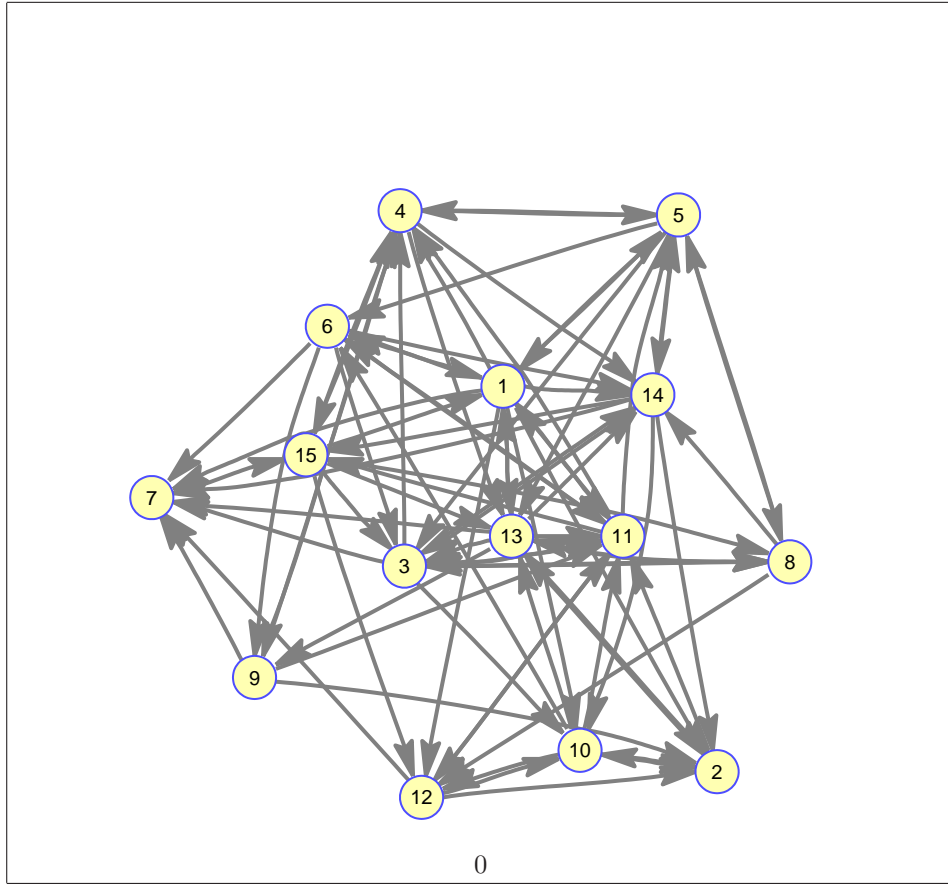
0

4-Assemblies



0

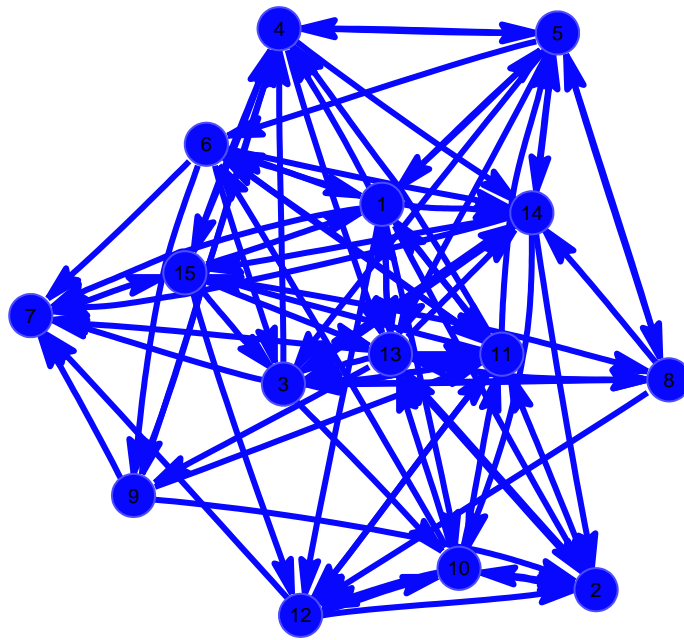
B.3.20 randomGraph(15,0.5,directed)-3



0	1	0	1	1	1	1	0	0	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	1	1	0	1	0	0
0	0	0	1	0	0	1	1	0	1	1	0	0	1	0
0	0	0	0	1	0	0	0	1	0	0	0	1	1	1
1	0	1	1	0	1	0	1	0	0	0	0	1	1	0
1	0	1	0	0	0	1	0	1	0	1	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	1	0	0	0	0	0	0	1	1	1	0
0	1	0	1	0	0	1	0	0	0	1	0	0	0	0
0	1	0	0	0	1	0	0	0	0	1	1	1	0	0
1	0	1	1	1	1	0	0	0	0	0	1	0	0	1
0	1	0	0	0	0	1	0	0	1	1	0	0	0	0
1	1	1	0	0	0	1	1	1	0	1	0	0	1	0
0	1	1	0	1	0	1	0	0	1	0	0	0	0	1
1	0	1	1	0	0	0	1	0	0	0	1	1	0	0

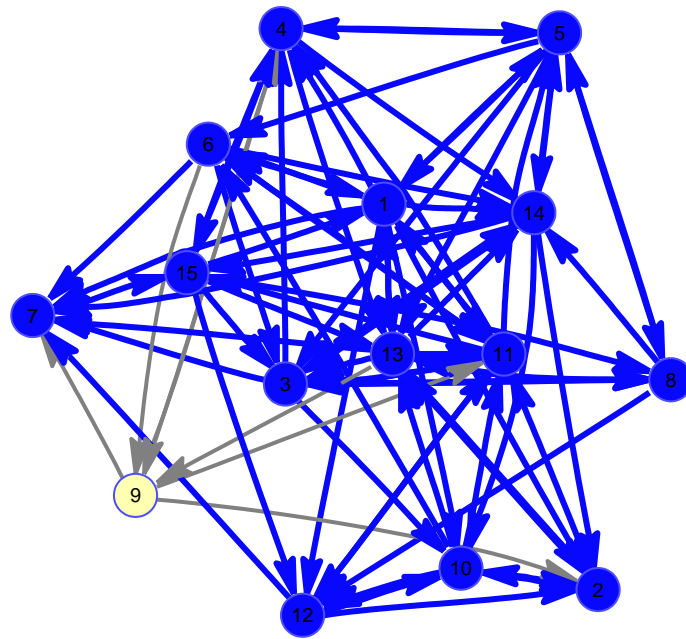
(65)

3-Assemblies



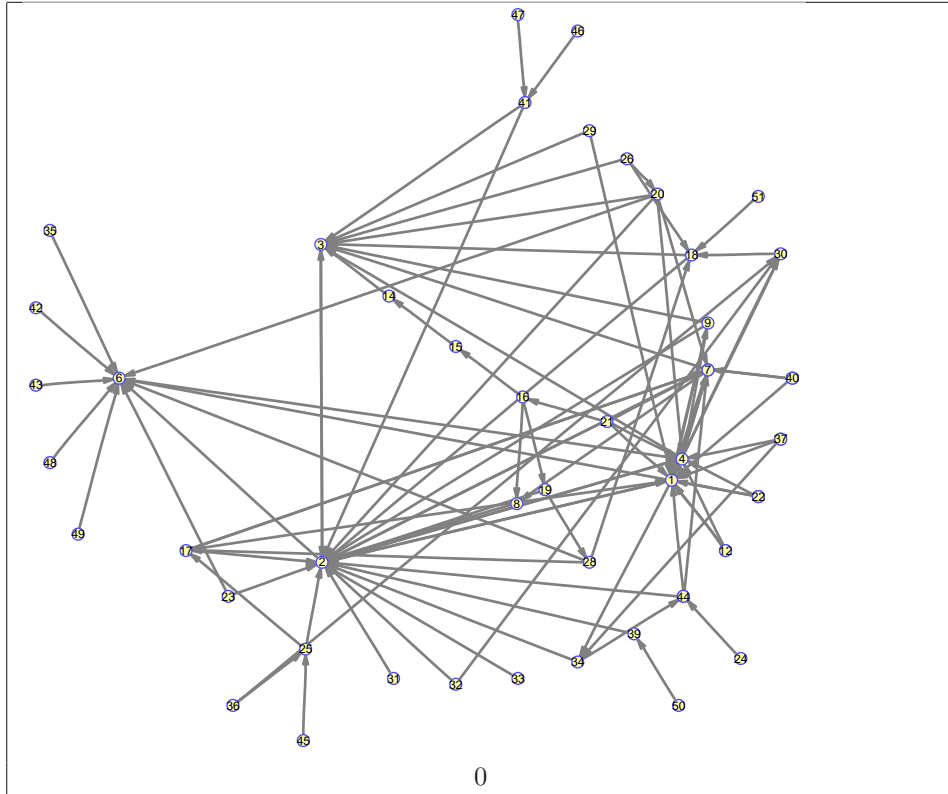
0

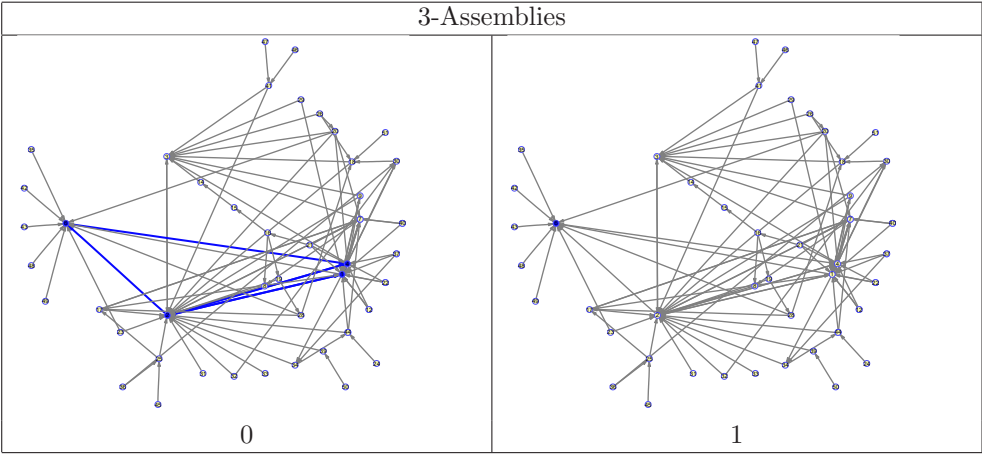
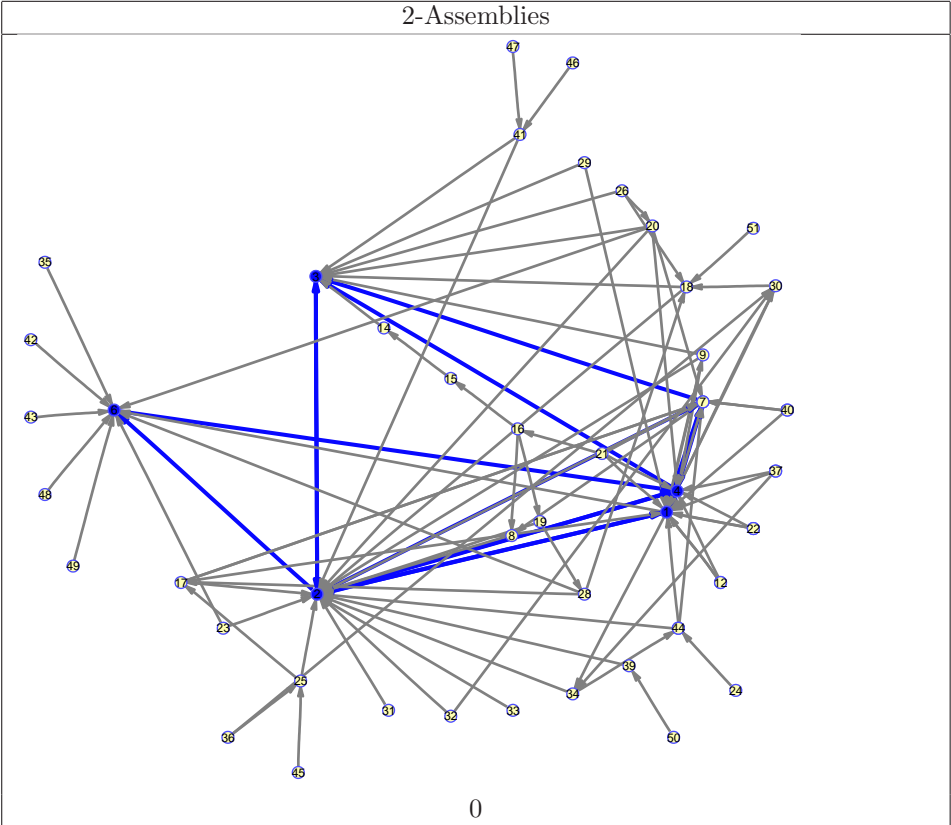
4-Assemblies

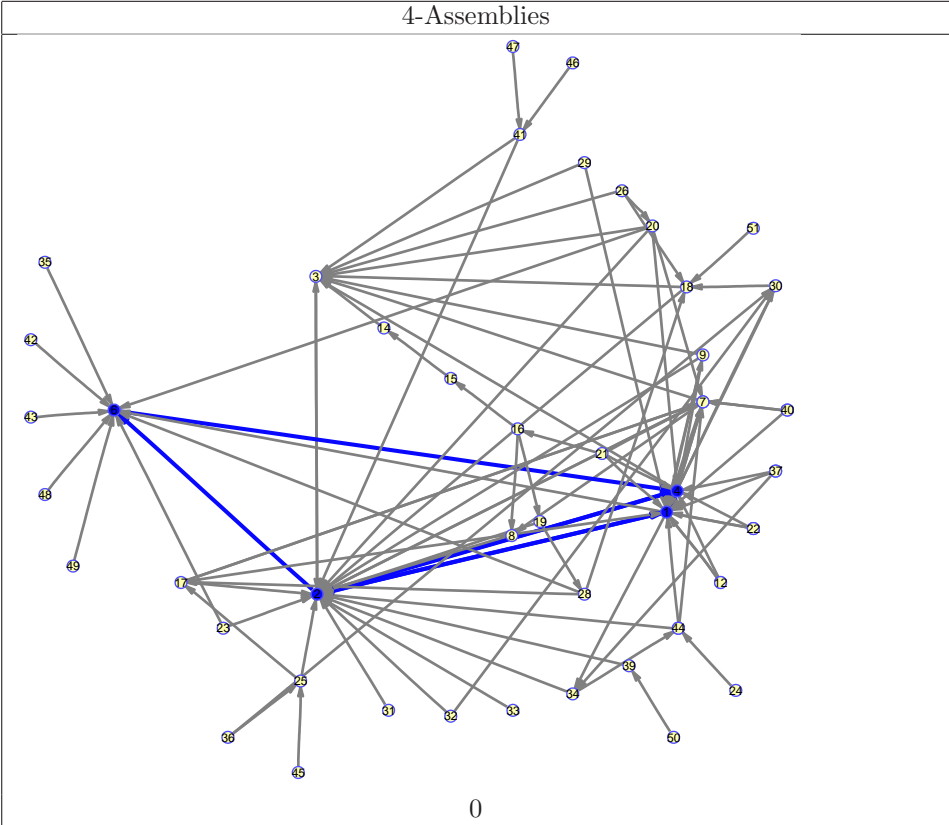


0

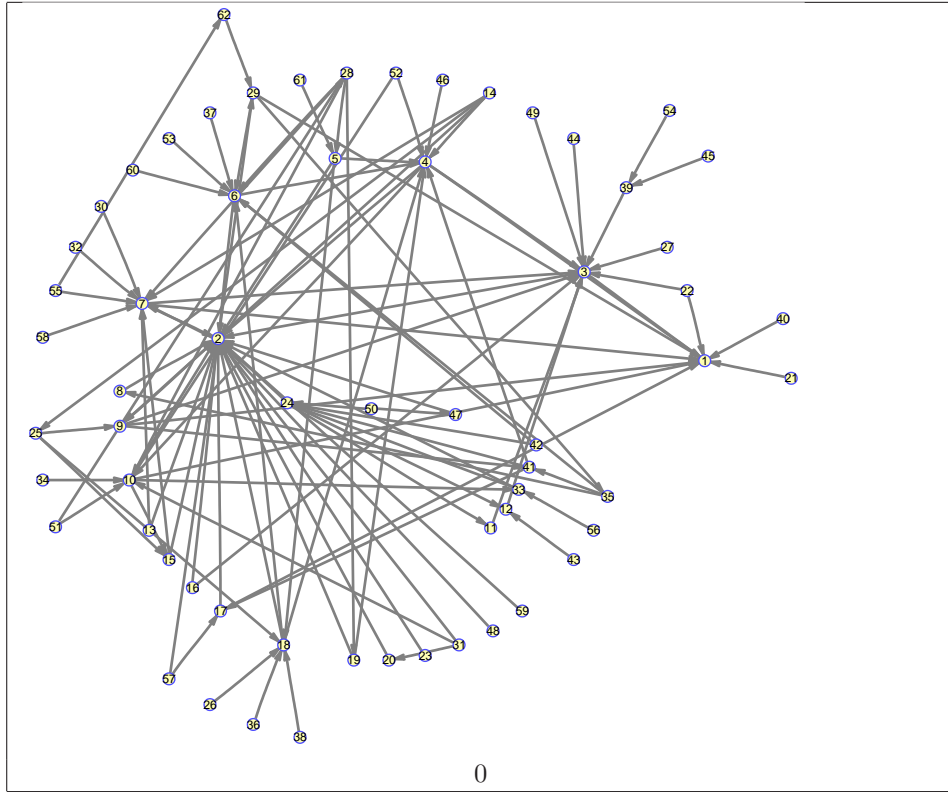
B.3.21 scaleFreeGraph(100,0.5,0,0,1,[[0.8,0.2],[0.8,0.2],directed)-1

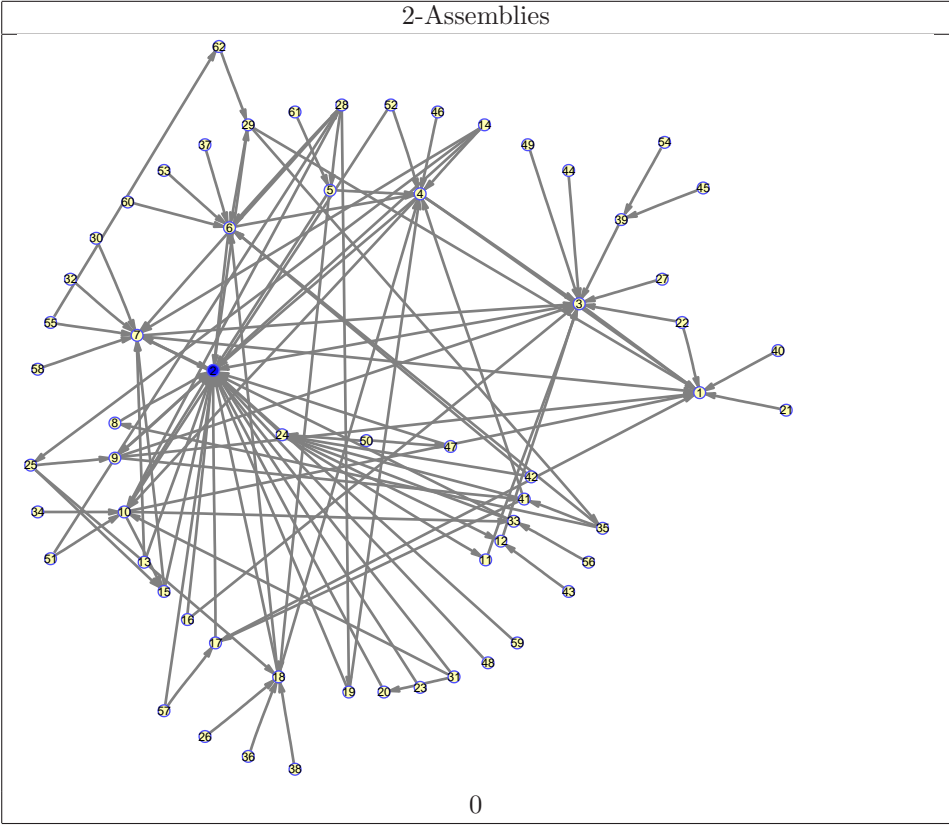




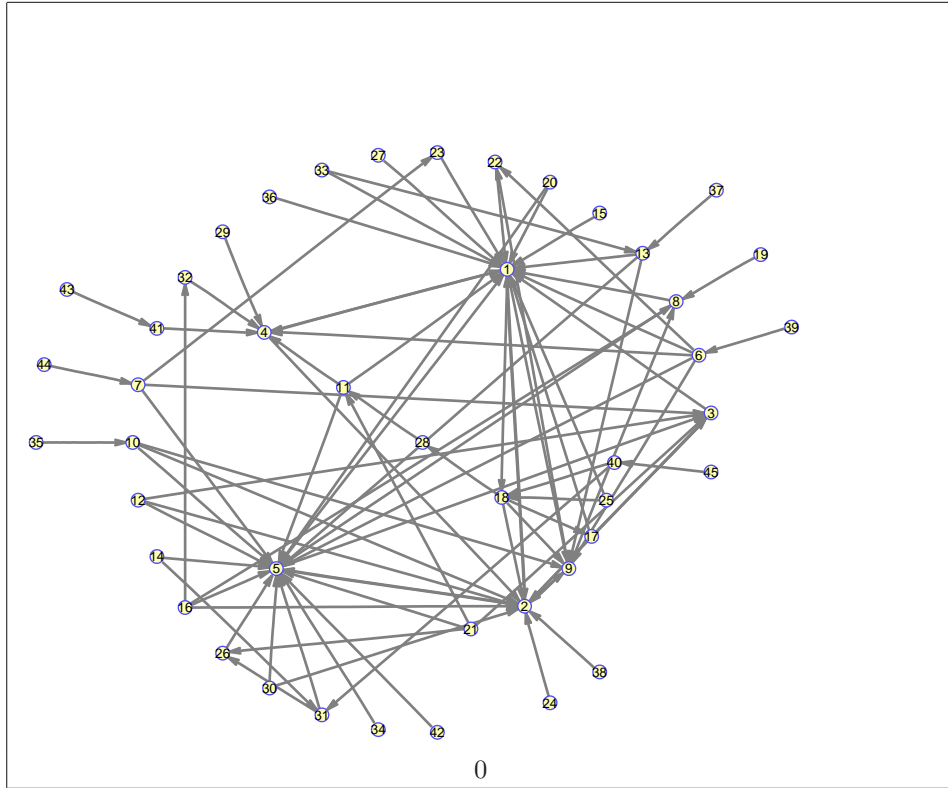


B.3.22 scaleFreeGraph(100,0.5,0,0,1,[[0.8,0.2],[0.8,0.2],directed)-2

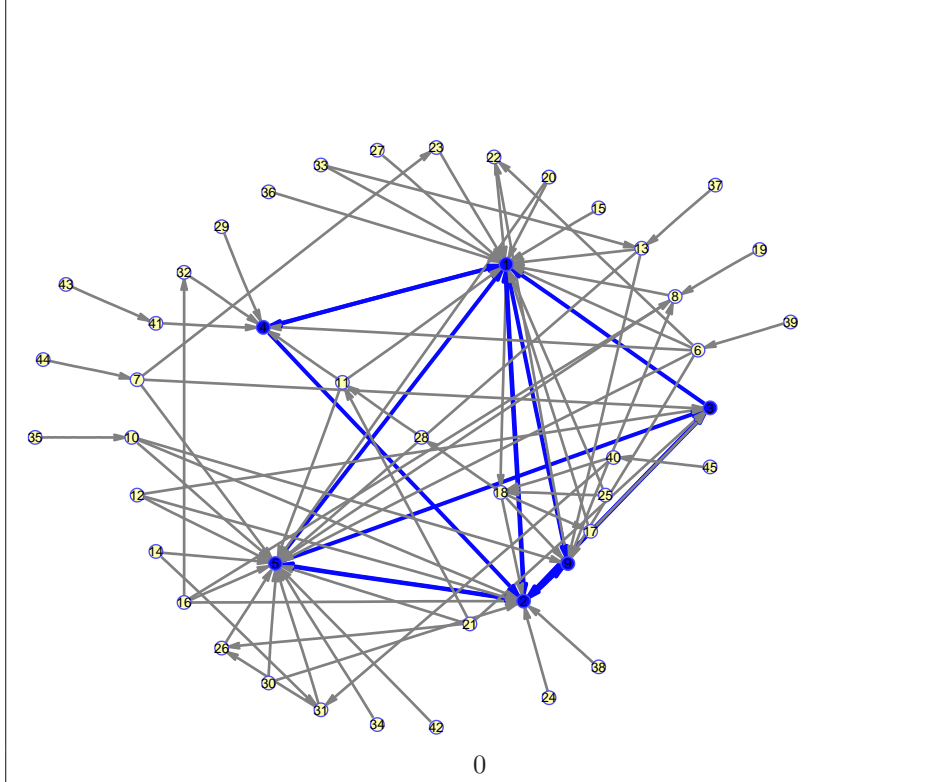




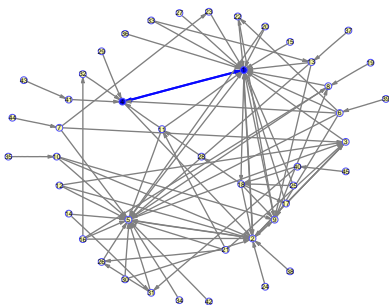
B.3.23 scaleFreeGraph(100,0.5,0,0,1,[[0.8,0.2],[0.8,0.2],directed)-3



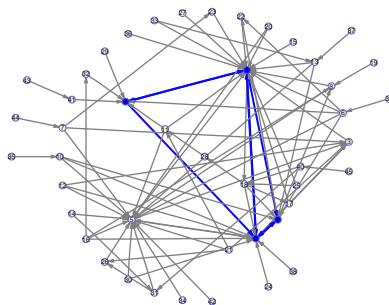
2-Assemblies



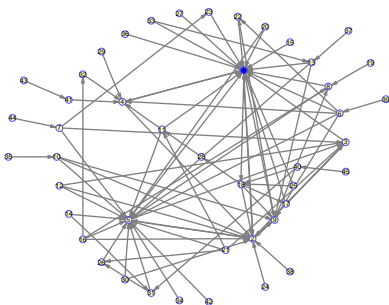
3-Assemblies



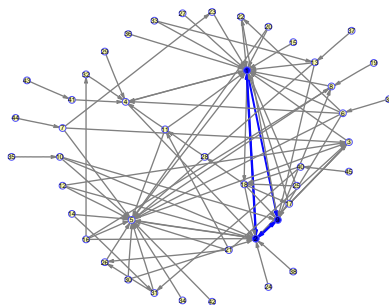
0



1

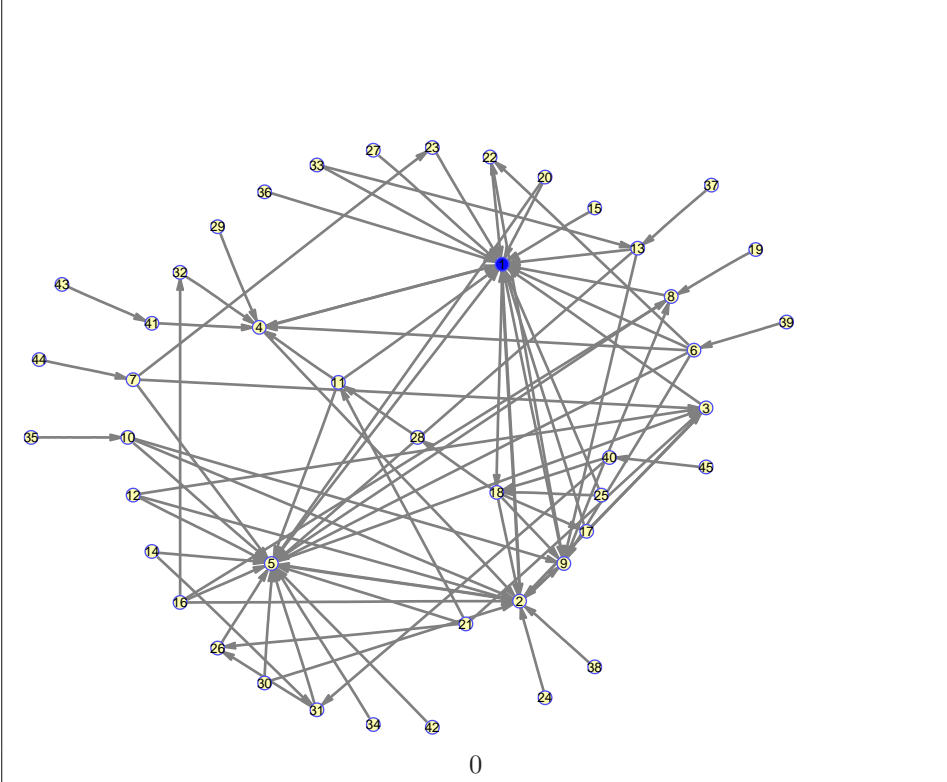


2

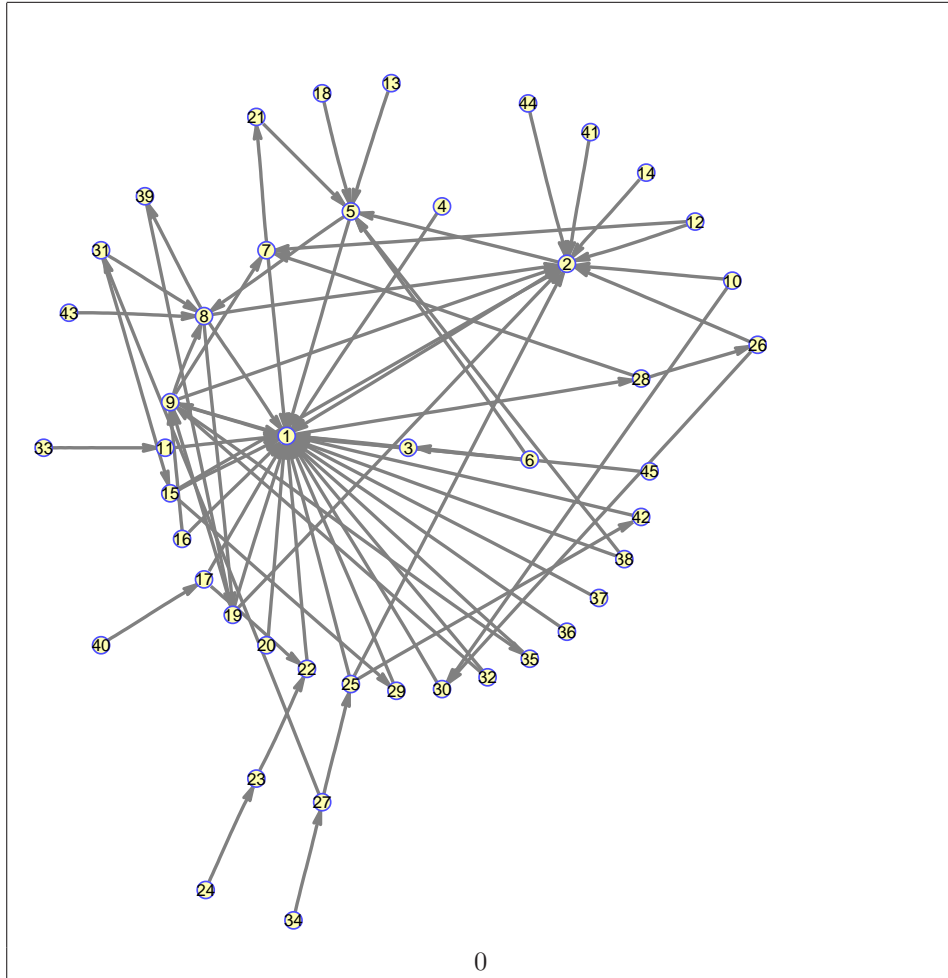


3

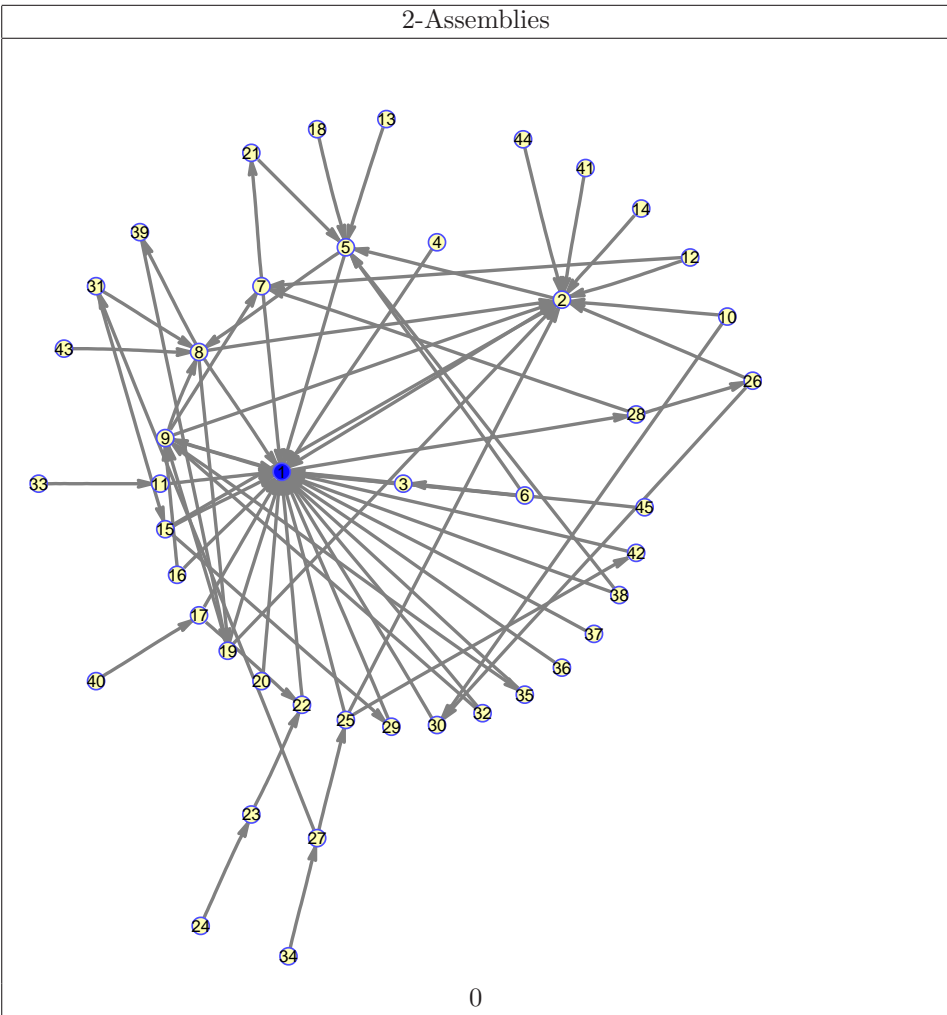
4-Assemblies



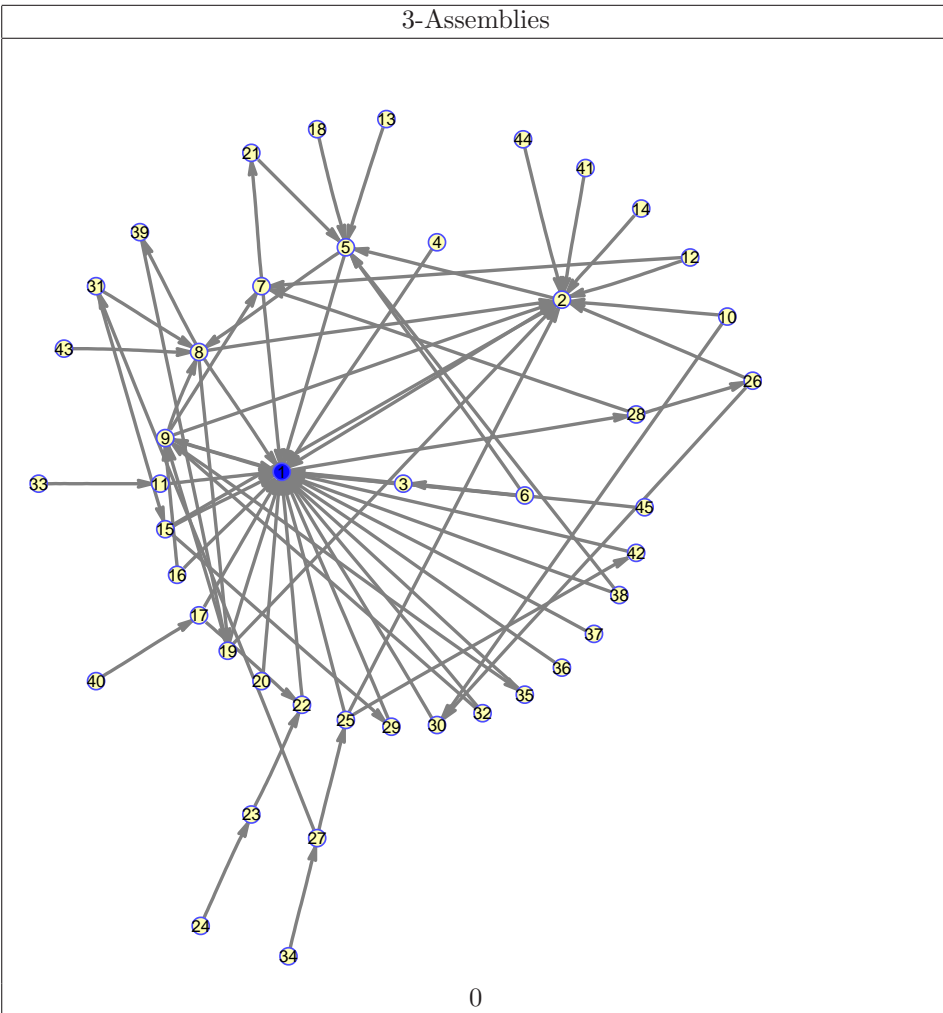
B.3.24 scaleFreeGraph(100,0.5,0,0,1,[[1],[1],directed)-1



2-Assemblies

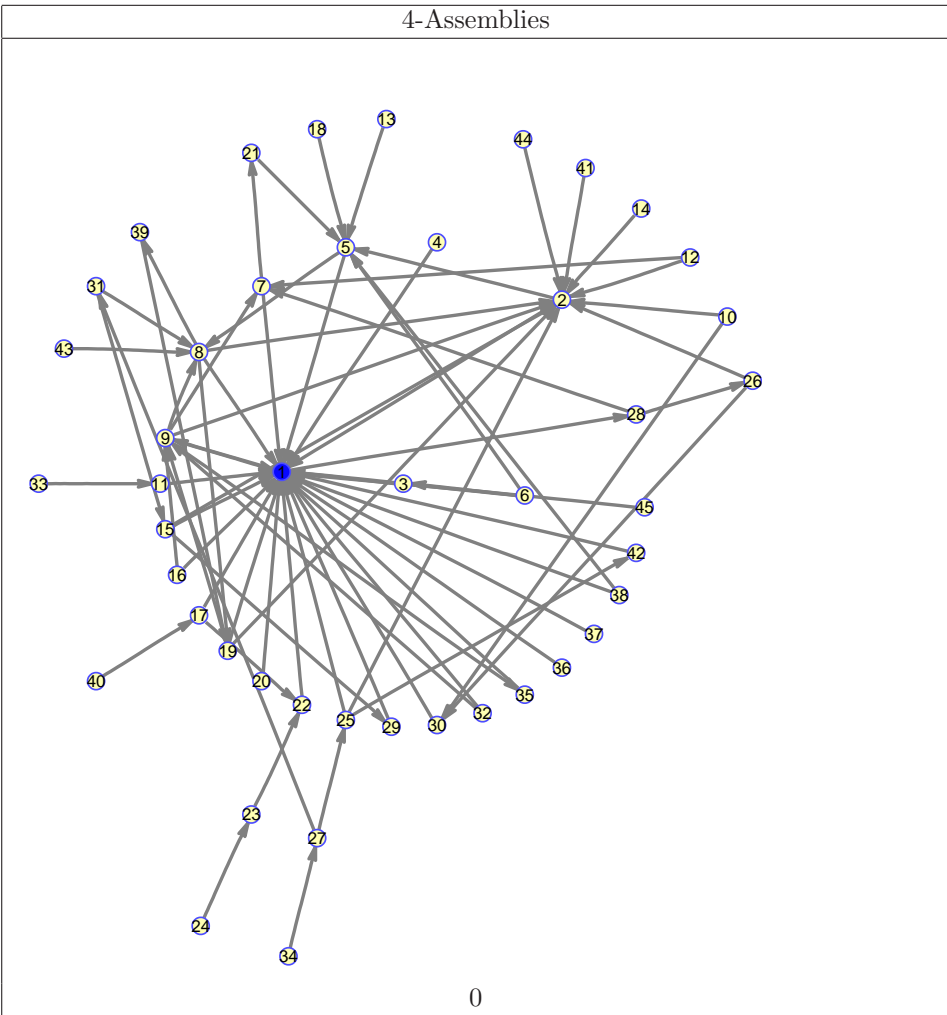


3-Assemblies

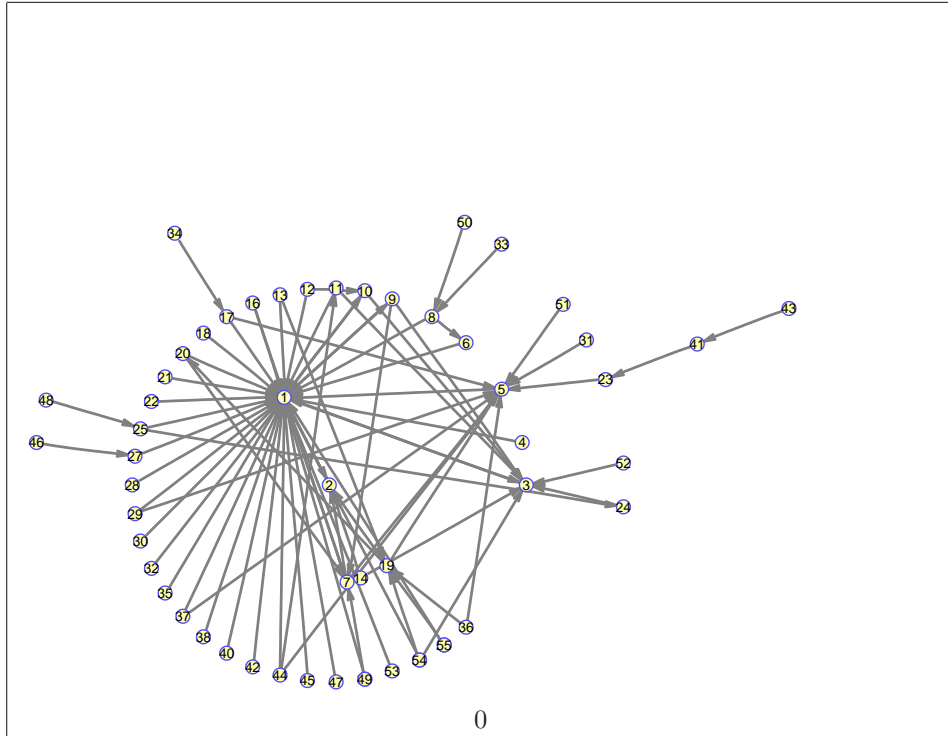


0

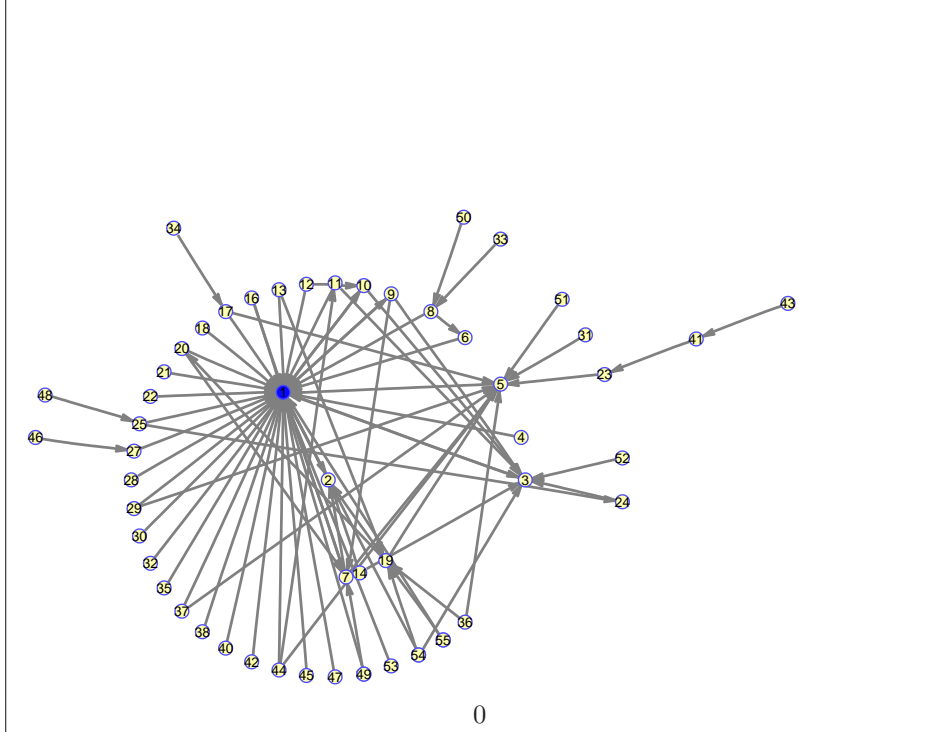
4-Assemblies



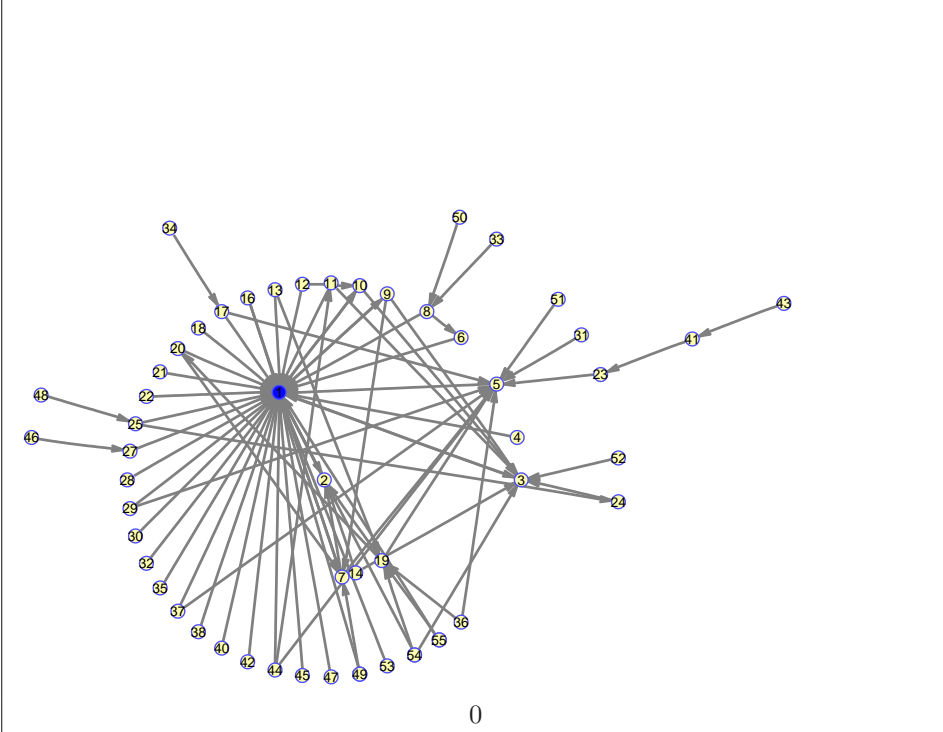
B.3.25 scaleFreeGraph(100,0.5,0,0,1,[[1],[1],directed)-2



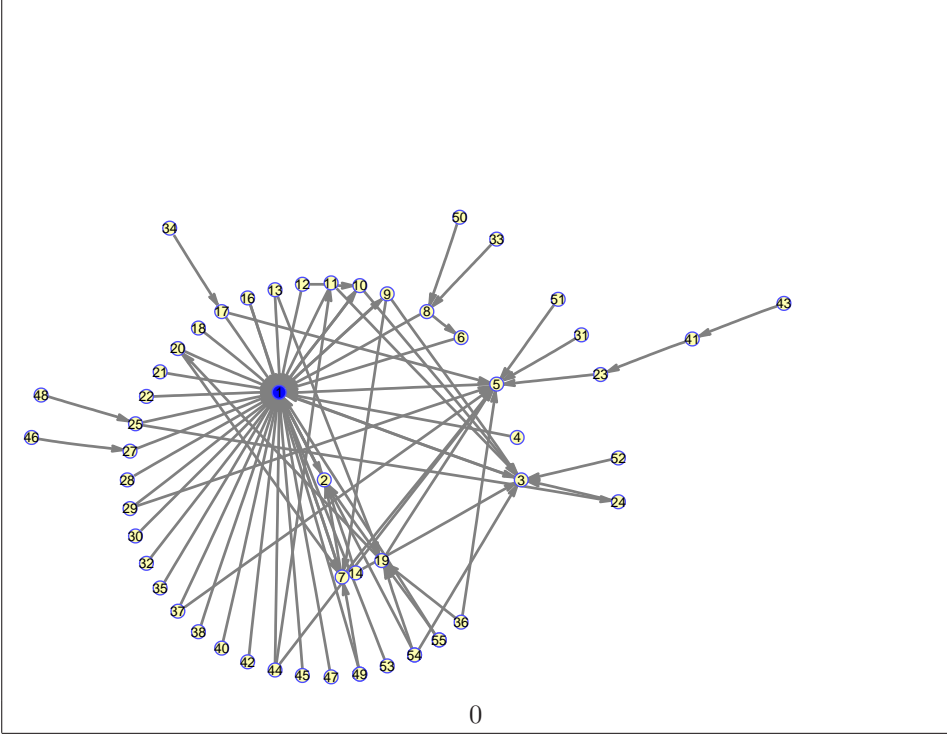
2-Assemblies



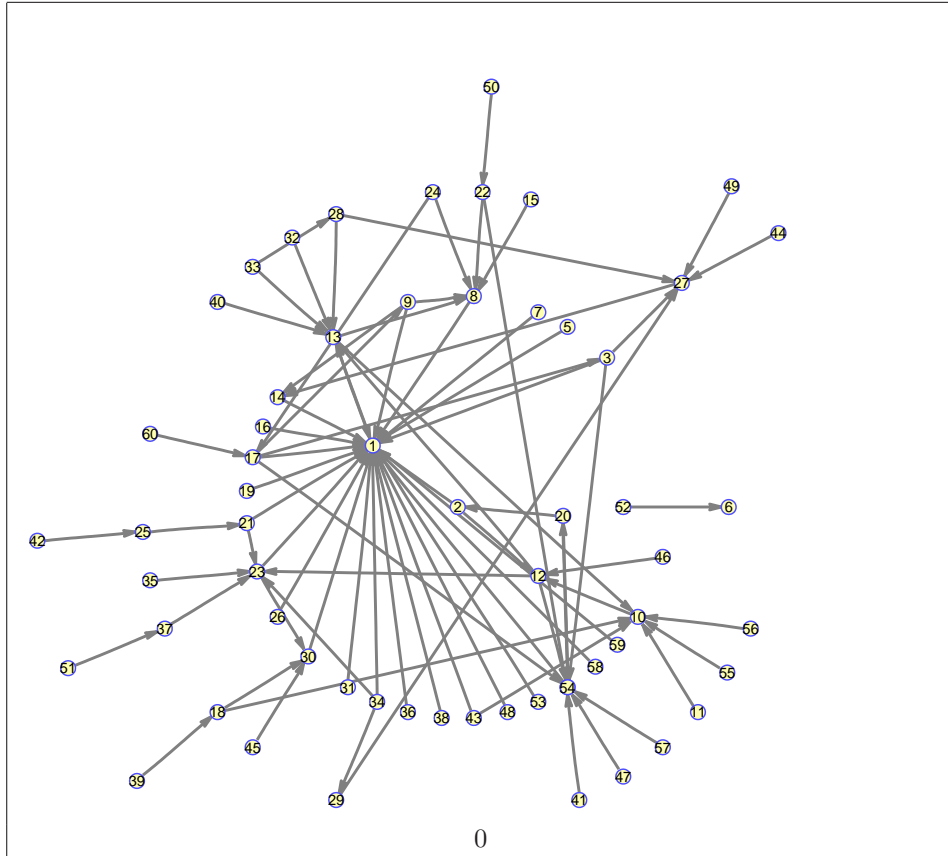
3-Assemblies



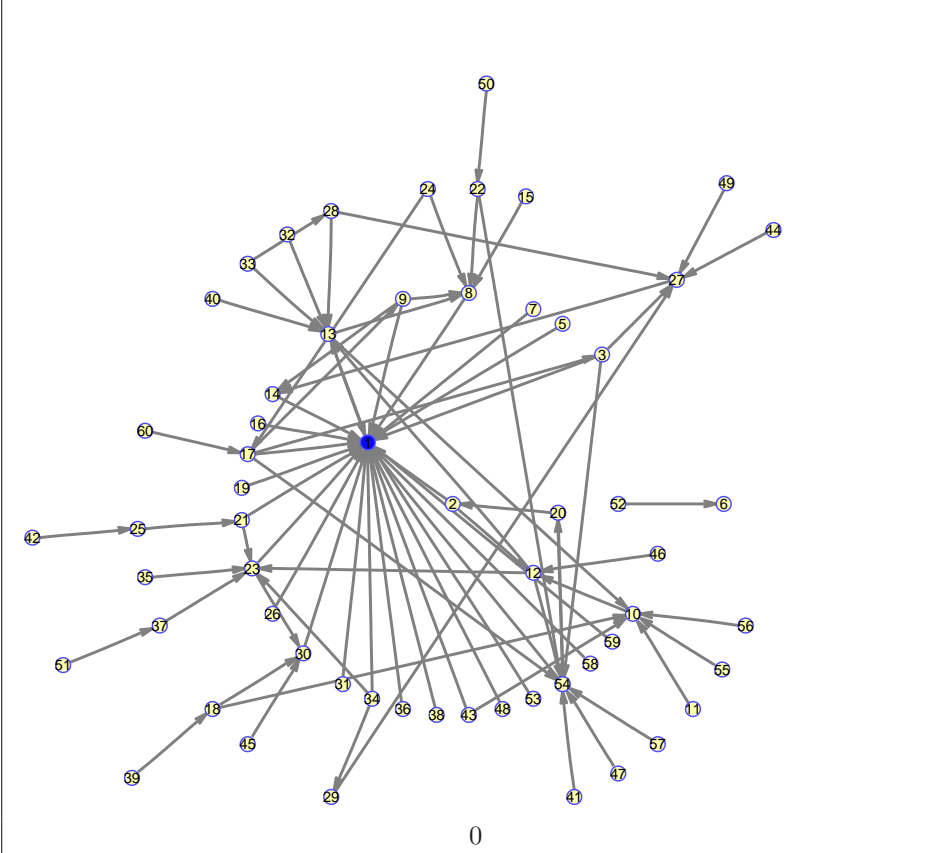
4-Assemblies



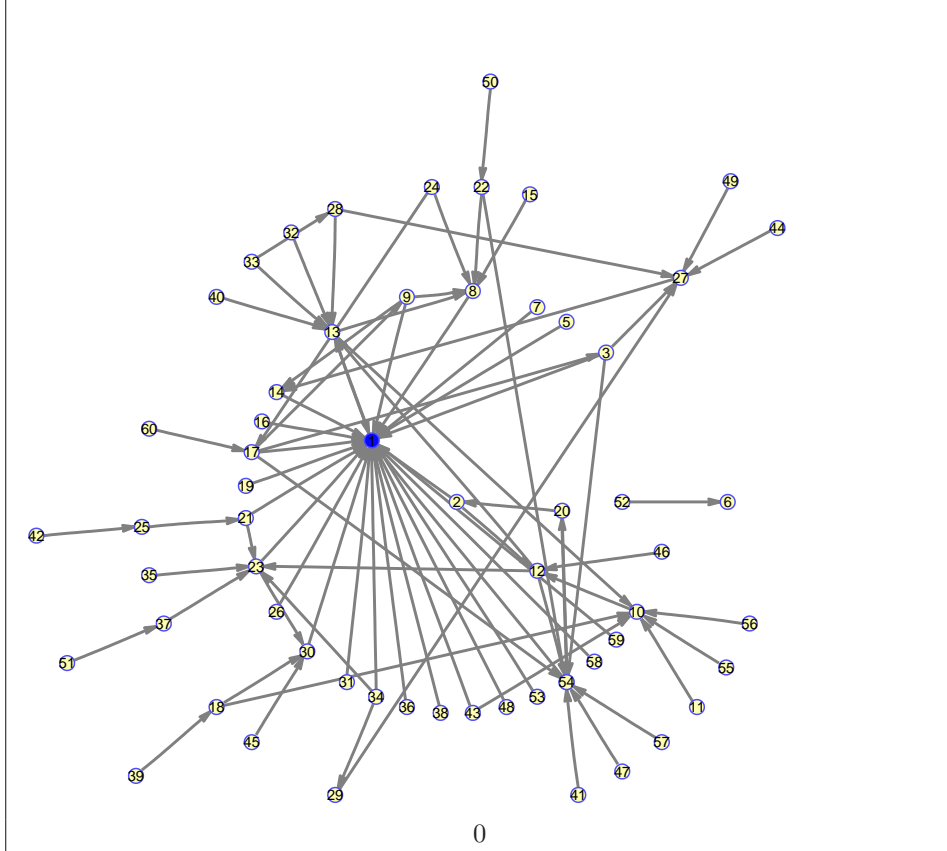
B.3.26 scaleFreeGraph(100,0.5,0,0,1,[[1],[1],directed)-3



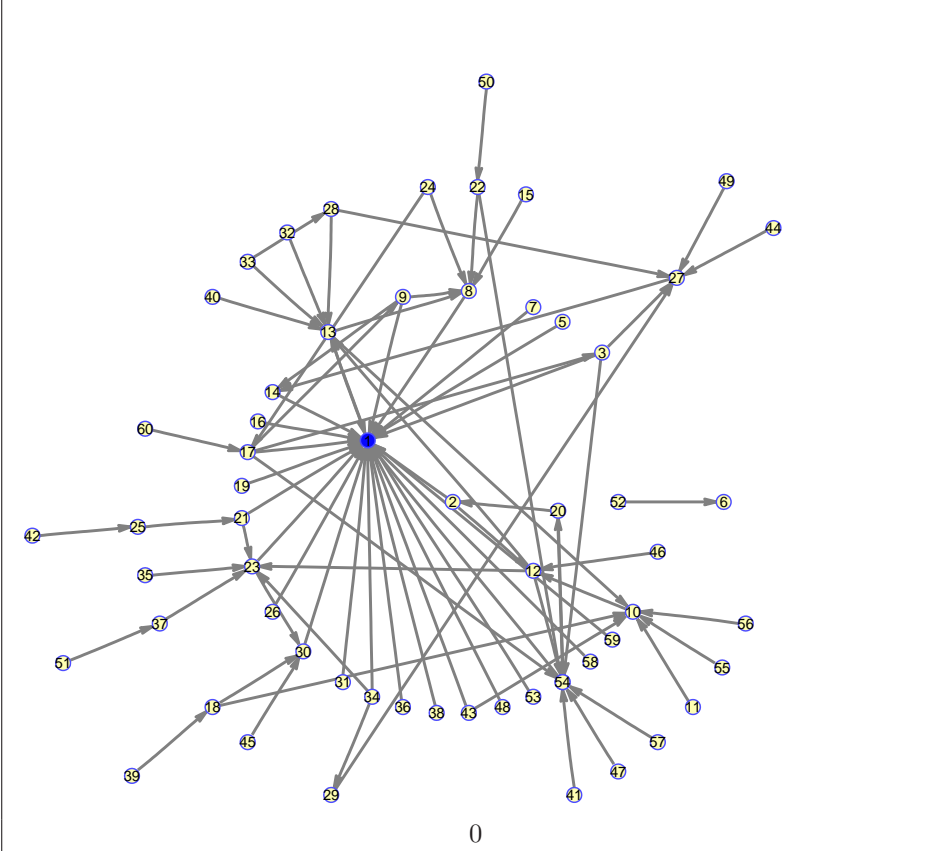
2-Assemblies



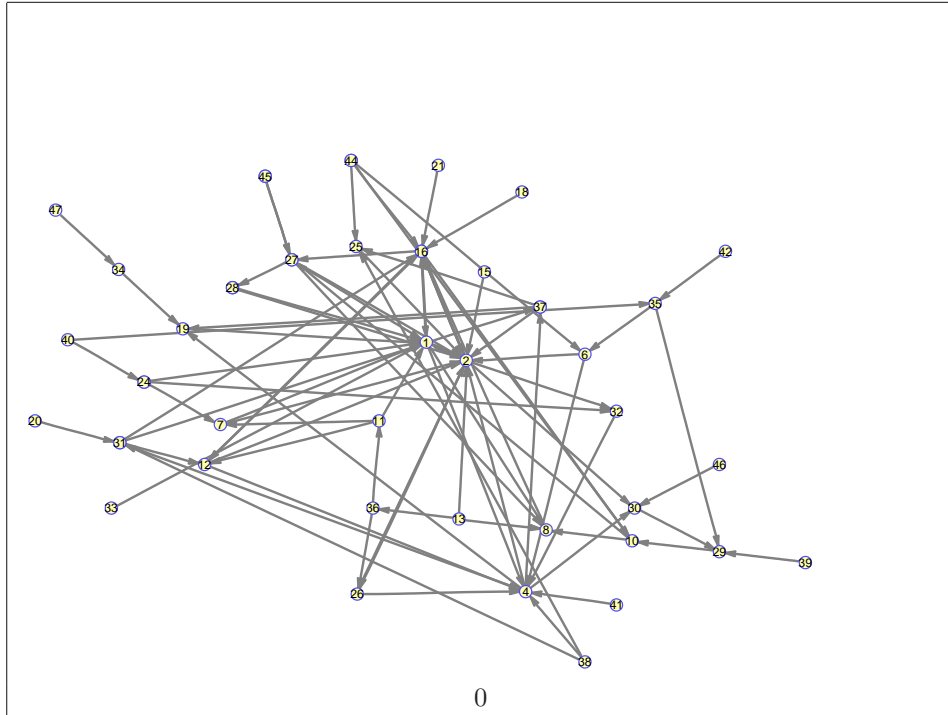
3-Assemblies



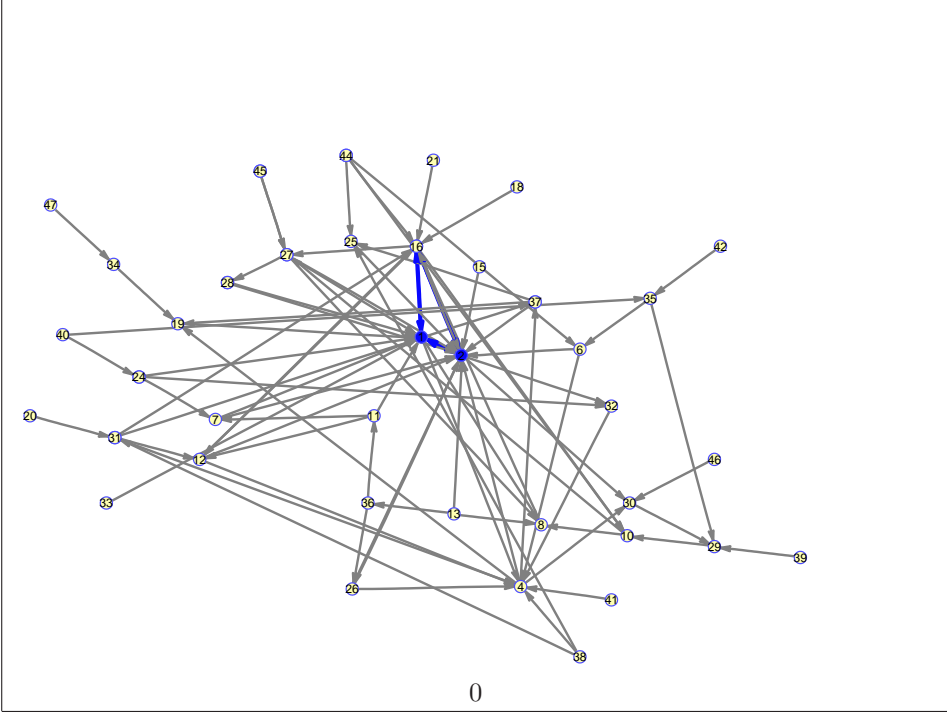
4-Assemblies



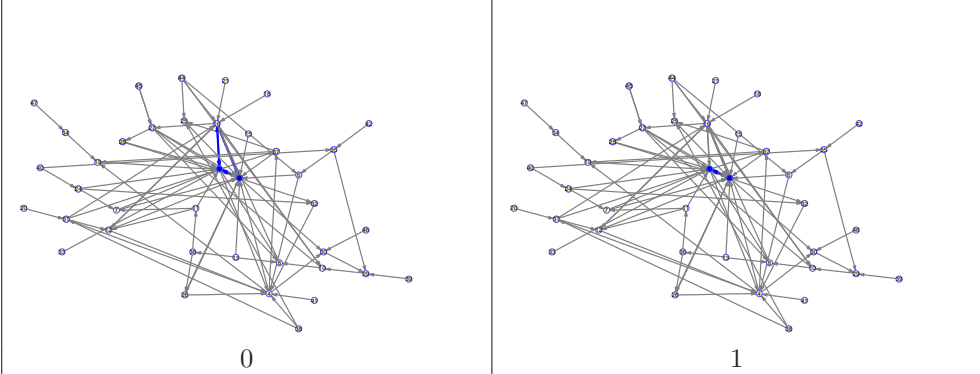
B.3.27 scaleFreeGraph(100,0.5,0.5,0.5,0.5,[[1],[1],directed)-1



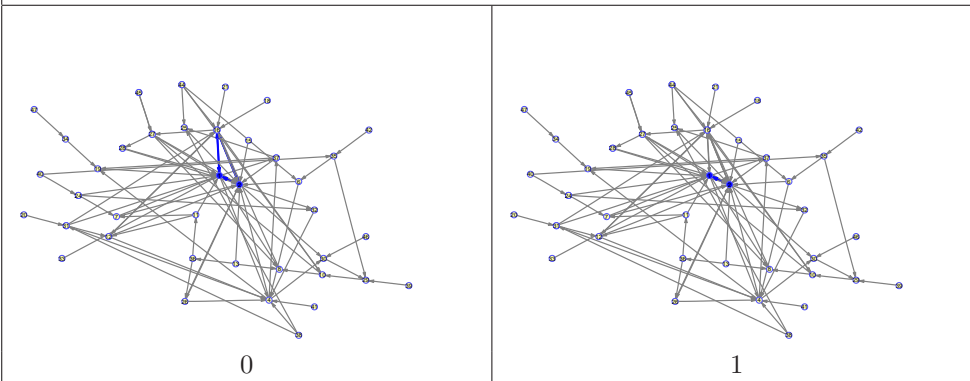
2-Assemblies



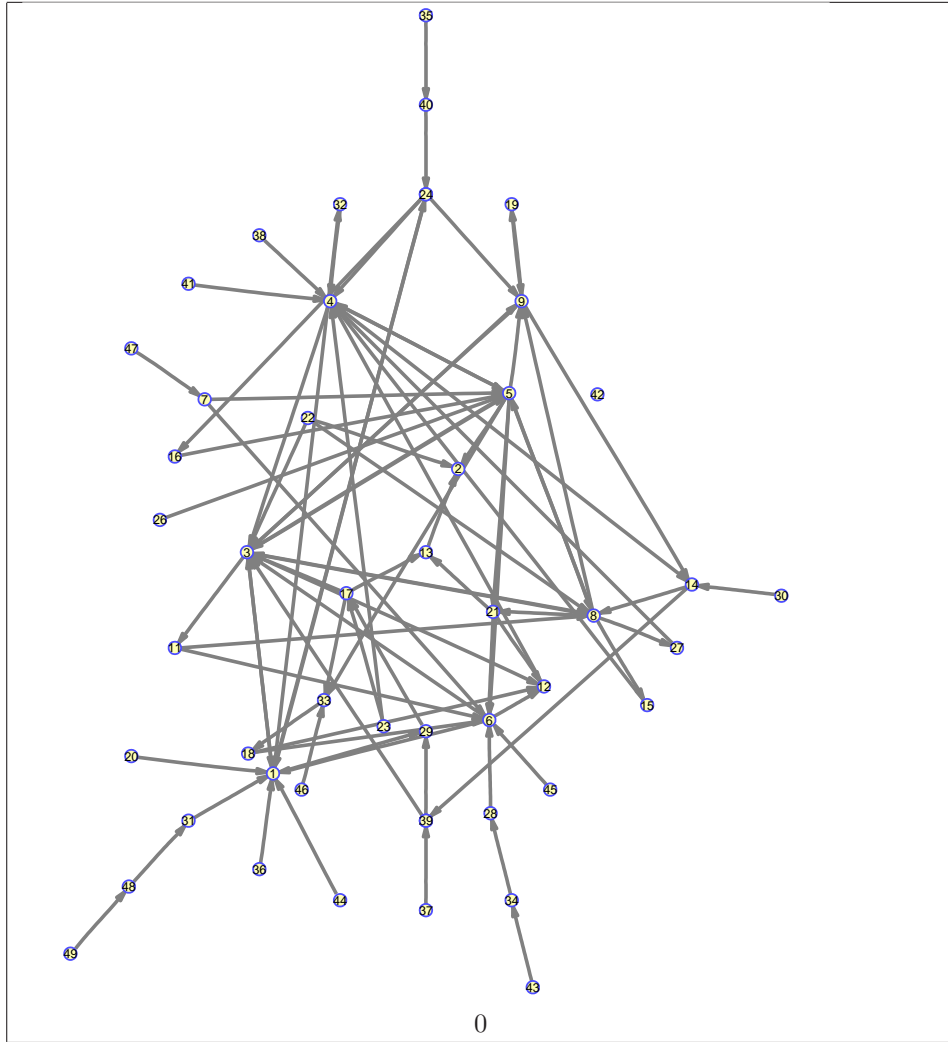
3-Assemblies

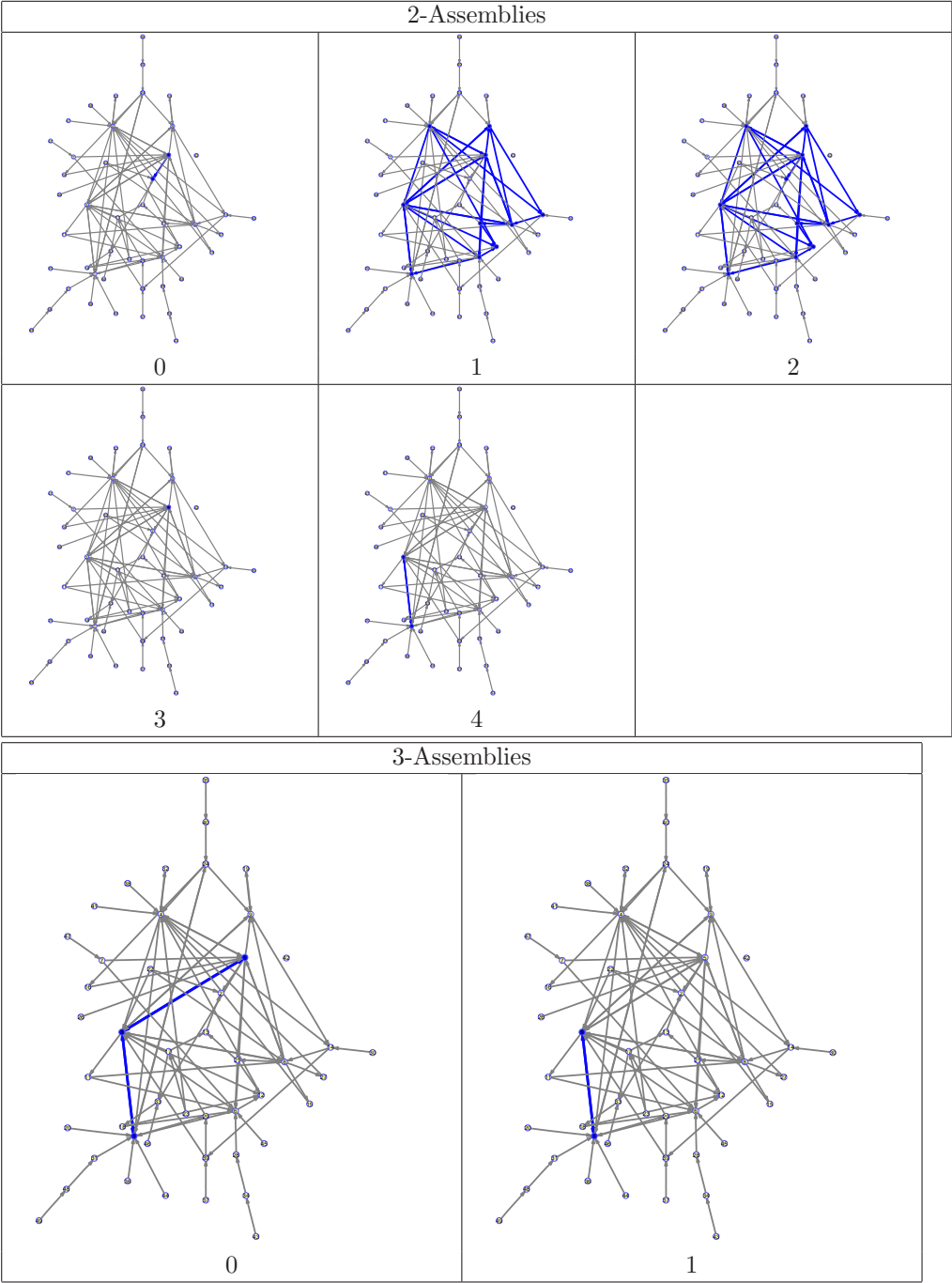


4-Assemblies

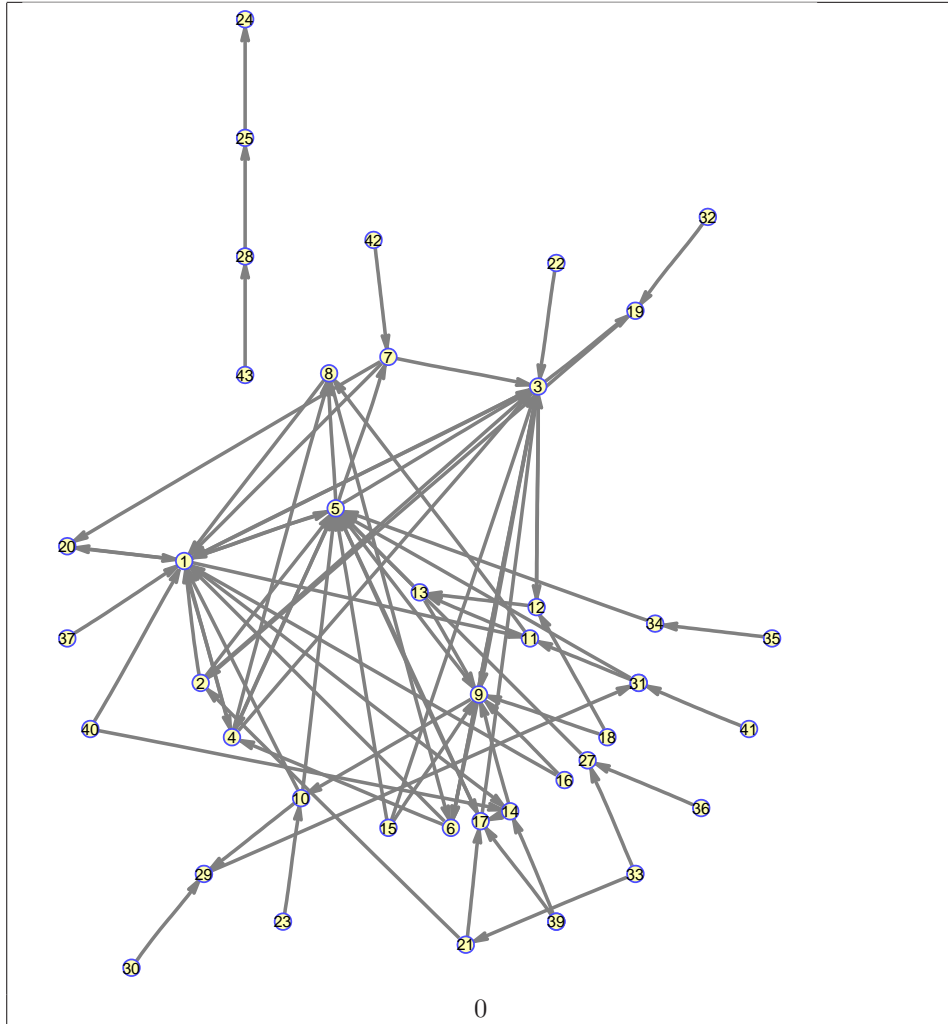


B.3.28 scaleFreeGraph(100,0.5,0.5,0.5,0.5,[[1],[1],directed)-2

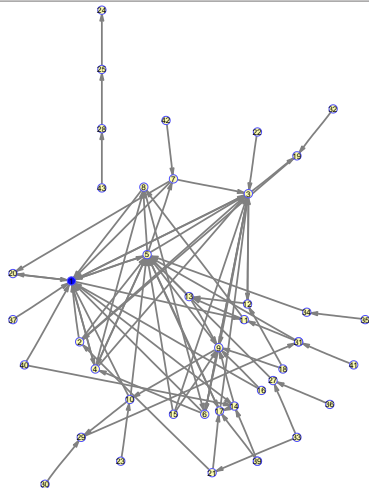




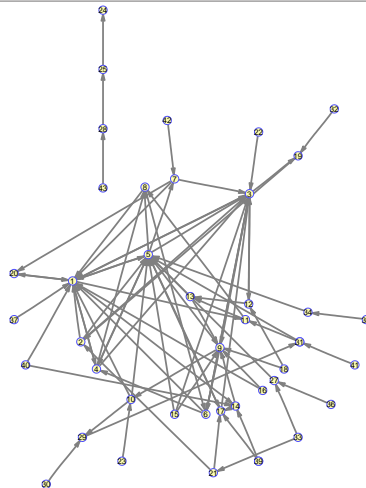
B.3.29 scaleFreeGraph(100,0.5,0.5,0.5,0.5,[[1],[1],directed)-3



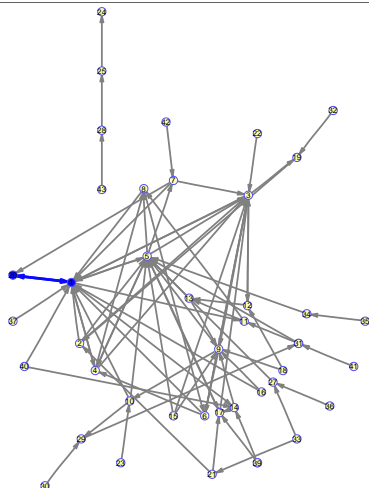
2-Assemblies



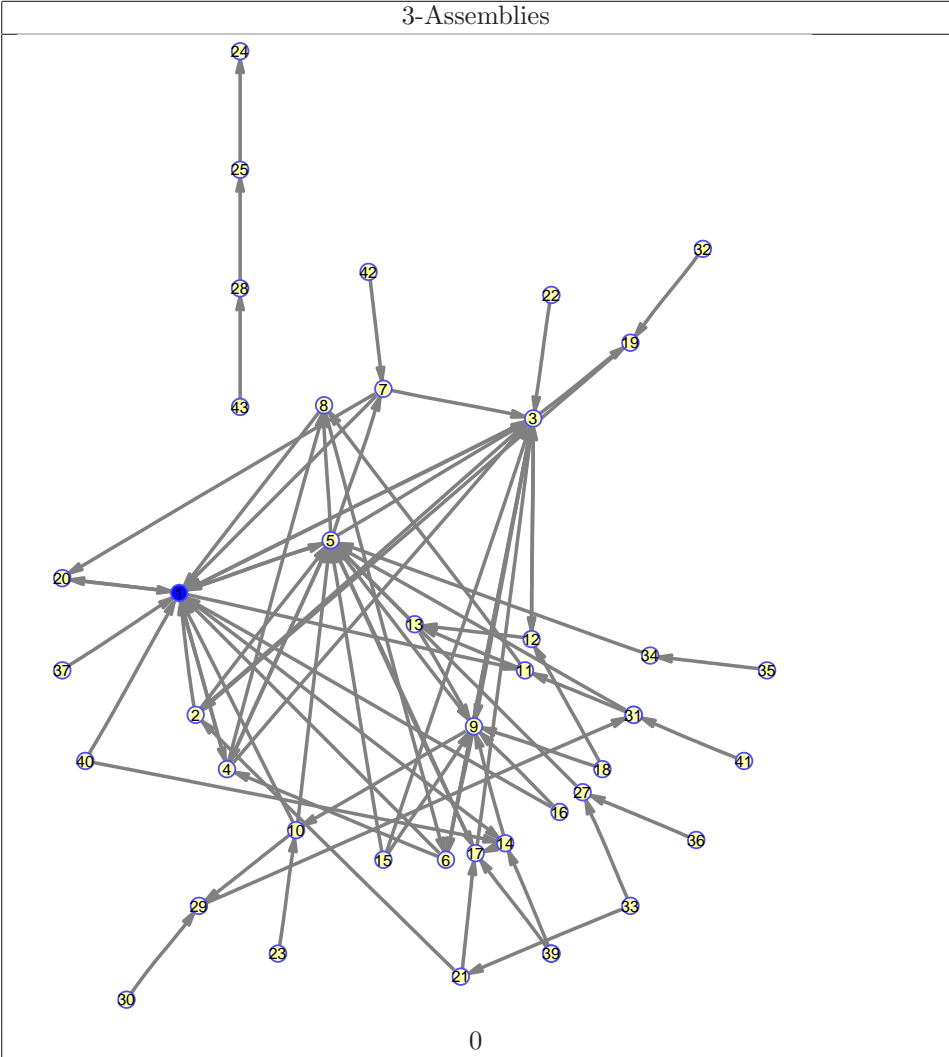
0

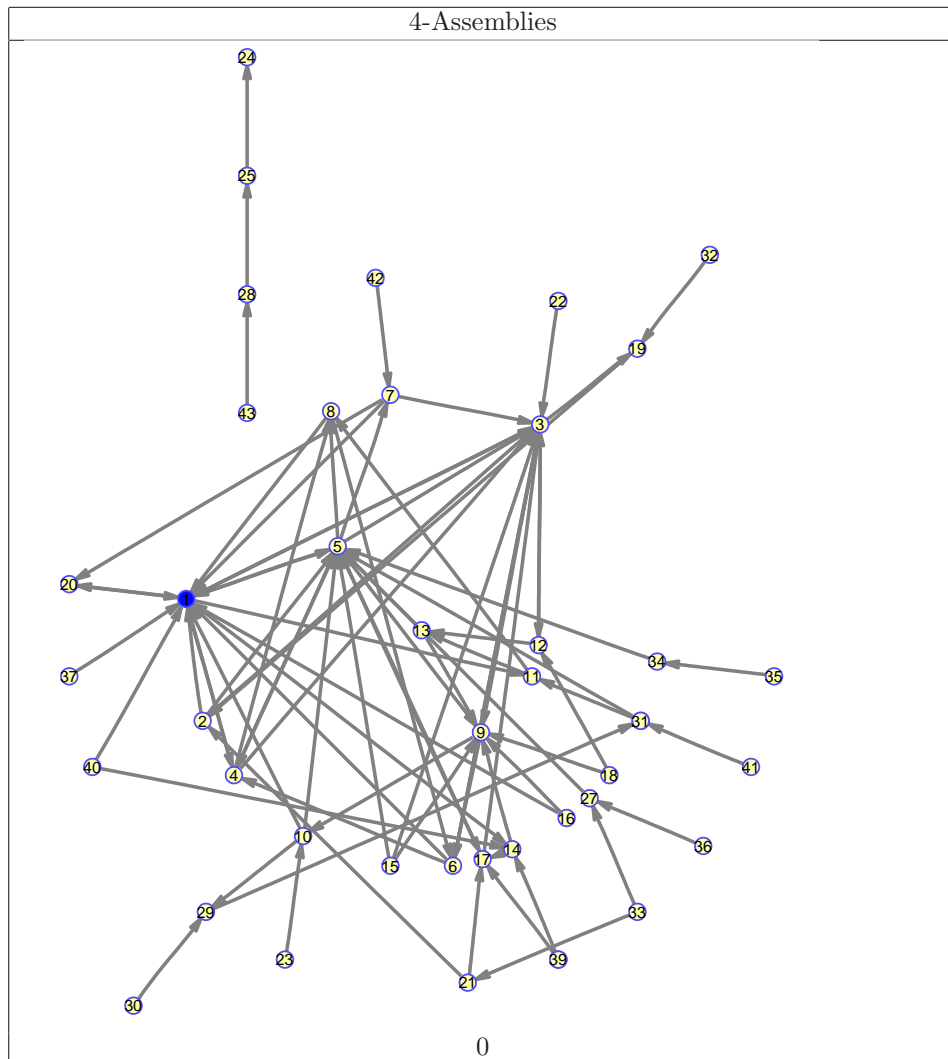


1



2





C Implementation

C.1 getCellAssemblies.m

```
% Khalif Halani
%
% getCellAssemblies.m
%
% usage: s = getCellAssemblies(A,coresize)
% where: A is the adjacency matrix of the graph
```

```

%         coresize is the size of the core you want to check
%
% the function returns a cell assembly, s, that consists of the cell assemblies
% found in the graph
function s = getCellAssemblies(A,coresize)
s = {};
[r, tr, cc] = enumCores(A,coresize);
[clset cltree]= closuretree(A,r,tr,cc,coresize);
for i = 1:length(clset)
    x = clset{i};
    if ~isempty(x)
        for j = 1:length(x(1,:))
            w = x{2,j};
            verified = 0;
            for k = 1:length(w(:,1))
                rindex = [w(k,1) w(k,2)];
                if checkTight(A,rindex,coresize,r,tr)
                    verified = 1;
                    break
                end
            end
            if verified == 1
                g = clset{i}{1,j};
                s = [s {g}];
            end
        end
    end
end
end
end

```

C.2 enumCores.m

```

% Zachary Rubenstein, Jordon Cavazos
%
% enumCores.m
%
% Finds all k-cores in a given graph for an arbitrary k as well as the
% sub-k-cores for each k-core.
%
% usage: r = enumCores(graph, coresize)
%
% whereas:
%   graph is an adjacency matrix
%   coresize is the annihilation threshold
%
%   r is a 1xN cell array, where N is the number of vertices in the

```

```

% largest k-core in the graph. Entry x of this array contains a
% another cell array, which in turn contains a number of
% vectors representing every sub k-core in the graph with x vertices,
% one vector per k-core. The contents of each vector is the indices
% contained in that k-core, numbered according to the given A.
%
% tr is a cell array laid out identically to r, except that, instead of
% a vector for each k-core, each k-core is represented by a Mx2
% matrix, where M is the number of sub k-cores inside the represented
% k-core. Each row in the matrix represents one of these sub k-cores
% the two columns in each row are the first and second elements of
% the location of that core in r. For example, say tr{5}{2} returns
% the matrix [3 2;4 1]. This means that the second core enumerated
% with five vertices has two sub k-cores. The first has three
% vertices, and its indices are located at r{3}{2}. The second has
% four vertices, and its indices can be found at r{4}{1}
%
% cc is a Px2 matrix, where P is the number of k-cores in the graph. Each
% row represents a single k-core. The first and second columns of
% the row represent the k-core's location in r and tr
%
% usage example:
%
% A = [0 1 1 1 0 0 0 0 0
% 1 0 1 1 0 0 0 0 0
% 1 1 0 1 0 0 0 0 0
% 0 0 0 1 0 1 1 1 1
% 0 0 0 0 1 0 1 1 1
% 0 0 0 0 1 1 0 1 1
% 0 0 0 0 1 1 1 0 1
% 0 0 0 0 1 1 1 1 0]
%
% [r tr cc] = enumCores(A, 3)

```

```
function [r, tr, cc] = enumCores(graph, coresize)
```

```

global resid_set
global tree
global corecount
nn=length(graph);
resid_set={};
corecount=[];

```



```

resid_set{nn} = {1:nn};
corecount=[nn 1];
tree=cell(1,nn);
enumHelp(graph, coresize, 1:length(graph));

r = resid_set;
tr= tree;
cc=corecount;
end

function copy=enumHelp(graph, coresize, activenodes)
copy=[];
loc=[];
global resid_set
global tree
global corecount
if isempty(activenodes)
    return
end

leng = length(activenodes);
initial=size(corecount,1);
for i = 1:leng
    resid = findAnnihilateComp(graph, coresize, activenodes, i);
    if not(isempty(resid))
        %cnctd= connected(graph,resid);
        %if cnctd
        nn=length(resid);
        bin=resid_set{nn};
        [temploc tf]=isin(bin, resid);
        temploc=[nn temploc];
        if not(tf)
            bin = [bin {resid}];
            if iscell(bin)
                gg=length(bin);
            else
                gg=1;
            end
            corecount=[corecount; [nn,gg]];
            resid_set{nn}=bin;
            tempcopy=enumHelp(graph, coresize, resid);
            copy=[copy; tempcopy];

elseif not(ismember(temploc,loc,'rows'))
    loc=[loc; temploc];

```

```

        end
        %end
    end
end
for j=1:size(loc,1)
    bin=tree{loc(j,1)};
    copy=[copy; loc; bin{loc(j,2)}];
end
copy=unique(copy,'rows');
tbin=tree{corecount(initial,1)};
tbin{corecount(initial,2)}=unique([corecount(initial+1:size(corecount,1),:);copy],'rows');
tree{corecount(initial,1)}=tbin;
end

%
% Yan Zhou, Zach Rubenstein
%
% findAnnihilateSet.m
%
% This function returns the annihilation set in the order that we
% annihilated them and its complement and a mutation of the original
% graph with all edges in the annihilation set removed.
%
function f = findAnnihilateComp(A, k, activenodes,...
    startNodeIndex)
f = activenodes;
r = startNodeIndex;
while not(or(isempty(r),isempty(f)))
    f(r) = [];
    r = find(sum(A(f,f)) < k);
end
end
%Vector of neighbor indices for node
function n = N(node, A)
n = find(A(node,:));
end
%Degree of node
function d = D(node, A)
d = sum(A(node,:));
end

function [loc tf] = isIn(array, target)
loc=[];
tf = 0;
if isempty(target)
    tf = 1;

```

```

        return
    end
    if isempty(array)
        tf=0;
        return
    end
    if not(iscell(array))
        if all(array == target)
            tf=1;
            loc=1;
            return
        end
    else
        for cellind = 1:length(array)
            curvector = array{cellind};
            if all(curvector == target)
                tf = 1;
                loc = cellind;
                return
            end
        end
    end
end
end

```

C.3 closuretree.m

```

%Jordon Cavazos
%
%closuretree.m
%
%Finds the closure of all k-cores in a graph.
%
%usage: [clset cltree]=closuretree(A,r,tr,cc,k)
%
%Where:
% A is the adjacency matrix
% k is the annihilation threshold
% r, tr, and cc are the outputs of function enumCores3.m
%
% cltree is a cell array identical in shape to r and tr. The index
%   cltree{cc(i,1)}{cc(i,2)} returns the closure of the ith k-core in
%   the cc table.
%
% clset is a cell array of unique closures. It is organized by the order
%   of the closure. clset{i} will return a 2xn cell array, where n is
%   the number of unique closures of order i. clset{i}{1,ii} will

```

```

%      return the iith closure of order i. clset{i}{2,ii} will return a
%      matrix where each row contains the lookup indices (much like cc)
%      of k-cores that has the iith closure of order i.
%
%usage example:
% A = [0 1 1 0 0 0 0 0 0 0 1 0;
%      1 0 0 1 0 0 0 0 0 0 0 1;
%      1 0 0 1 1 0 0 0 0 0 0 0;
%      0 1 1 0 0 1 0 0 0 0 0 0;
%      0 0 1 0 0 1 1 0 0 0 0 0;
%      0 0 0 1 1 0 0 1 0 0 0 0;
%      0 0 0 0 1 0 0 1 1 0 0 0;
%      0 0 0 0 0 1 1 0 0 1 0 0;
%      0 0 0 0 0 0 1 0 0 1 1 0;
%      0 0 0 0 0 0 0 1 1 0 0 1;
%      1 0 0 0 0 0 0 0 1 0 0 1;
%      0 1 0 0 0 0 0 0 0 1 1 0];
%
% [r, tr, cc] = enumCores3(A, 2);
% [clset cltree]=closuretree(A,r,tr,cc,2)

function [clset cltree]=closuretree(A,r,tr,cc,k)
n=length(A);
nc=size(cc,1);
clset={};
clset{n}={};
cltree={};
cltree{n}={};

for j=1:nc      %for every k-core
    activenodes=r{cc(j,1)}{cc(j,2)}; %activate k-core for closure
    c=getClosure(A,activenodes,k); %close
    cltree{cc(j,1)}{cc(j,2)}=c; %add closure to tree
    nn=length(c); %find length for hashing
    if isempty(clset{nn})
        c1={c;cc(j,:)};
        clset{nn}=c1;
    else
        [loc tf]=isin(clset{nn}(1,:),c); %check to see if already found
        if tf==0
            c1={c;cc(j,:)};
            clset{nn}=[clset{nn} c1];
        else
            clset{nn}{2,loc}=[clset{nn}{2,loc};cc(j,:)];
        end
    end
end

```

```

        end
    end

    end

    % isIn
    %
    % Finds whether or not vector is in the set of already found vertices. If
    % so, gives location.
    function [loc tf] = isIn(array, target)
    loc=[];
    tf = 0;
    if isempty(target)
        tf = 1;
        return
    end
    if isempty(array)
        tf=0;
        return
    end
    if not(iscell(array))
        if all(array == target)
            tf=1;
            loc=1;
            return
        end
    else
        for cellind = 1:length(array)
            curvector = array{cellind};
            if all(curvector == target)
                tf = 1;
                loc = cellind;
                return
            end
        end
    end
end
end
end

```

C.4 checkTight.m

```

% Khalif Halani
%
% checkTight3D.m
%
% checks whether a given k-core is k-tight

```

```

%
% usage: tf = checkTight3D(A,rindex,coresize,r,tr)
% where: A is the adjacency matrix of the graph
%         rindex is the index of the core in r
%         coresize is the size of the core you want to check
%         r is the list of k-cores
%         tr is the tree of k-cores
%
% the function returns either a 1 or a 0 to indicate whether the k-core is
% k-tight(1) or not(0)
function tf = checkTight(A,rindex,coresize,r,tr)
kcore = r{rindex(1)};
kcore = kcore{rindex(2)};
flag = checkMin(A, kcore, coresize);
if flag == 1
    tf = 1;
    return
end
if flag == 2
    tf = 0;
    return
end
x = tr{rindex(1)};
x = x{rindex(2)};

sleng = length(x(:,1));
tf = 1;
for i = 1:sleng
    if tf==1
        s = r{x(i,1)};
        s = s{x(i,2)};
        C = getClosure(A, s, coresize);
        comp = getComp(kcore,s);
        cl = getClosure(A, comp, coresize);
        if (checkSubset(C,kcore)==0 && ~isempty(cl))
            tf=0;
        end
    end
end
return
end

```

C.5 checkMin.m

```

% Khalif Halani

```

```

%
% checkMin.m
%
% checks whether a given k-core is minimal
%
% usage: tf = checkMin(A, kcore, coresize)
% where:   A = the adjacency matrix of the graph
%          kcore = the vector of node numbers that make up the kcore
%          coresize = the minimum connections for a core
%
% the function returns either a 1 or a 0 to indicate whether the core was
% minimal(1) or not(0)

function tf = checkMin(A, kcore, coresize)
tf = 1;
for i = 1:length(kcore)
    if tf==0
        return
    end
    d = sum(A(:,kcore(i)));
    if d<coresize
        tf=2;
        return
    end
    s = findAnnihilateComp(A,coresize,kcore,i);
    if isempty(s)
        tf = 1;
    else tf = 0;
    end
end
return
end

function f = findAnnihilateComp(A, k, activenodes,...
    startNodeIndex)
    f = activenodes;
    r = startNodeIndex;
while not(or(isempty(r),isempty(f)))
    f(r) = [];
    r = find(sum(A(f,f)) < k);
end
end

```

C.6 getClosure.m

```
% getClosure.m
%
% Kalif Halani, derived from Steve Cox and Tyler Young, kcore2.m
% (http://cnx.org/content/m30936/latest/)
%
% finds the closure of an arbitrary set in an arbitrary graph
%
% example: r = getClosure(A,kcore,k)
%   where:
%       r is a vector of indices in the closure in relation to A
%       A is the adjacency matrix of the graph
%       kcore is a vector of vertex indices for which the closure will be
%           found.
%       k is the activation threshold.
%
% example: r = getClosure([0 1 1;1 0 1;1 1 0],[1 2],1)

function r = getClosure(A,kcore,k)
N = length(A);
activenodes = zeros(N,1);
activenodes(kcore) = 1;
AO = A';
y = AO*activenodes;
y(y<k) = 0; y(y>=k) = 1;
ncnt = 1;
while norm(y-activenodes) > .5 && ncnt < N
    activenodes = y;
    y = AO*activenodes; y(y<k) = 0; y(y>=k) = 1;
    ncnt = ncnt + 1;
end
r = find(activenodes);
r = r';
return
end
```

C.7 getComp.m

```
% Khalif Halani
%
% getComp.m
%
% the function gives us the complement of a given set, B, in another set, A
%
% usage: comp = getComp(A, B)
```



```

% where:    A = a vector containing node numbers of the set we want to get
%           the complement from
%           B = a vector containing the node numbers of the set we want to
%           get the complement of
%
% the function returns a vector containing the node numbers of the
% complement of B in A
function comp = getComp(A,B)
comp = [];
for i = 1:length(A)
    if(isempty(find(B==A(i))))
        comp = [comp A(i)];
    end
end
return
end

```

C.8 checkSubset.m

```

% Khalif Halani
%
% checkSubset.m
%
% checks whether a given set is a subset of another set
%
% usage: tf = checkSubset(super, sub)
% where: super is the set that you want to check is a superset of the other
%        set
%        sub is the set that you want to check is a subset of the other set
%
% the function returns either a 1 or a 0 to indicate whether the set is a
% subset(1) or not(0)

function e = checkSubset(super, sub)
if isequal(super,sub)
    e=0;
    return
end
i = 1;
e = 1;
leng = length(sub);
while (e==1 && i<=leng)
    s = find(super == sub(i));
    if isempty(s)
        e = 0;
    else i=i+1;
end

```

```

    end
end
return
end

```

C.9 BRandom.m

```

% BRandom.m
%
% Zachary Rubenstein
%
% Generates the adjacency matrix for a Bernoulli random graph.
%
% usage A = BRandom(n, p, directed)
%   whereas:
%       n is the number of vertices in the graph
%       p is the probability that any given set of two vertices is
%         connected.
%       directed is a boolean variable: true if the graph to be generated is
%         directed, false if the graph is to be undirected.
%
% example: A = BRandom(100, .5, 1)

function A = BRandom(n, p, directed)

A = zeros(n,n);

if directed
    for node1 = 1:n
        for node2 = setdiff(1:n, node1)
            if rand < p
                A(node1, node2) = 1;
            end
        end
    end
else
    for node1 = 1:n-1
        for node2 = node1+1:n
            if rand < p
                A(node1, node2) = 1;
                A(node2, node1) = 1;
            end
        end
    end
end
end

```

end

C.10 CFRandom.m

```
% CFRandom.m
%
% Zachary Rubenstein
%
% Generates an adjacency matrix for a random, directed, scale-free graph by
% the Cooper-Frieze model (Bollobas 14,15).
%
% usage: A = CFRandom(T, alpha, beta, gamma, delta, p, q)
%
%   for the meaning of the arguments, refer to the reference.
%
% example: A = CFRandom(100, 0, 0, 0, 0, [.5 .5], [.5 .5])
%
% Reference:
%
% Bela Bollobas and Oliver Riordan. Mathematical results on scale-free
% random graphs. In Stefan Bornholdt and Heinz Georg Schuster, editors,
% Handbook of Graphs and Networks: From the Genome to the Internet, pages
% 132. Wiley, Weinheim, 2003.
```

```
function A = CFRandom(T, alpha, beta, gamma, delta, p, q, directed)
```

```
A = zeros(T+1,T+1);
connections = zeros(1,T+1);
connections(1) = 1;
nnodes = 1;
A(1,1) = 1;
cp = cumsum(p);
cq = cumsum(q);

for t = 1:T
    unifterm = 0;
    if rand < alpha
        % OLD
        nedgesvec = find(rand < cq);
        if rand < delta
            % initial node selected uniformly
            snode = ceil(rand*nnodes);
        else
            % initial node selected by degree
            cc = cumsum(connections);
```

```

        snodevec = find(rand*cc(end) < cc);
        snode = snodevec(1);
    end
    if rand < gamma
        unifterm = 1;
    end
else
    % NEW
    nnodes = nnodes + 1;
    nedgesvec = find(rand < cp);
    snode = nnodes;
    if rand < beta
        unifterm = 1;
    end
end
nedges = nedgesvec(1);
% if not(directed) % presumably, a node only gains degree
% when connections are incoming
if 1
    connections(snode) = connections(snode) + nedges;
end
es = zeros(1, nedges);
if unifterm
    for e = 1:nedges
        es(e) = ceil(rand*nnodes);
    end
else
    cc = cumsum(connections);
    for e = 1:nedges
        sedgevec = find(rand*cc(end) < cc);
        es(e) = sedgevec(1);
    end
end
if directed
    for e = 1:nedges
        enode = es(e);
        connections(enode) = connections(enode) + 1;
        weight = A(snode, enode);
        A(snode, enode) = weight + 1;
    end
else
    for e = 1:nedges
        enode = es(e);
        connections(enode) = connections(enode) + 1;
        weight = A(snode, enode);
        A(snode, enode) = weight + 1;
    end
end

```

```

        A(enode, snode) = weight + 1;
    end
end
end
A = A(1:nnodes,1:nnodes);
end

```

C.11 enumCores5.m

```

% Zachary Rubenstein, Jordon Cavazos
%
% enumCores5.m
%
% Finds all k-cores in a given graph for an arbitrary k as well as the
% sub-k-cores for each k-core. This differs from enumCores.m in that it
% uses knowledge of graph structure to improve performance. The main
% difference is the subfunction findAnnihilateComp2, which stops early if
% it was going to return a k-core already found.
%
% usage: r = enumCores5(graph, coresize)
%
% whereas:
%   graph is an adjacency matrix
%   coresize is the annihilation threshold
%
%   r is a 2xN cell array, where N is the number of k-cores in the
%   specified graph. Each column represents one core. The entry in the
%   first row of the column is the indices of the vertices (in relation
%   to the adjacency matrix) that compose the core. The second row of
%   the column is the indices of the columns in r that specify the
%   sub-k-cores of the core specified by the column.
%
% usage example:
%
%   A = [0    1    1    1    0    0    0    0    0
%        1    0    1    1    0    0    0    0    0
%        1    1    0    1    0    0    0    0    0
%        0    0    0    1    0    1    1    1    1
%        0    0    0    0    1    0    1    1    1
%        0    0    0    0    1    1    0    1    1
%        0    0    0    0    1    1    1    0    1
%        0    0    0    0    1    1    1    1    0]
%
% [r, tr, cc] = enumCores5(A, 3)

```

```

function [r, tr, cc] = enumCores5(graph, coresize)

global resid_set
global tree
global corecount
nn=length(graph);
resid_set={};
corecount=[];
resid_set{nn} = {1:nn};
corecount=[nn 1];
tree=cell(1,nn);
enumHelp(graph, coresize, 1:length(graph));

r = resid_set;
tr= tree;
cc=corecount;
end

function copy=enumHelp(graph, coresize, activenodes)
copy=[];
loc=[];
global resid_set
global tree
global corecount
if isempty(activenodes)
    return
end

ff=[]; %node to index of activenodes map (useful for findAnnihilateComp)
for j=1:length(activenodes)
    ff(activenodes(j))=j;
end

leng = length(activenodes);
initial=size(corecount,1);
%for i = 1:leng
count = 1;
vec=1:leng;
while count <= leng
    i=vec(count);
    if i==0
        count=count+1;
        continue
    end
end

```

```

end
[resid fc] = findAnnihilateComp(graph, coresize, activenodes, i,ff);
annset=vec(fc);
vec(fc)=0;
[copy loc]=enum(graph,coresize,copy,loc,resid);
count=count+1;
CI=i;
for j=2:length(annset)
    if annset(j)==0
        continue
    end
    [resid skip] = findAnnihilateComp2(graph, coresize, activenodes, annset(j),CI);
    if skip==1
        CI=[CI annset(j)];
        continue
    else
        [copy loc]=enum(graph,coresize,copy,loc,resid);
    end
end
end
for j=1:size(loc,1)
    copy=[copy; loc; tree{loc(j,1)}{loc(j,2)}];
end
copy=unique(copy,'rows');
tree{corecount(initial,1)}{corecount(initial,2)}=unique([corecount(initial+1:size(corecount,1),1)
end

```

```

function [copy loc]=enum(graph,coresize,copy,loc,resid)
global resid_set
global corecount
if not isempty(resid)
    %cnctd= connected(graph,resid);
    %if cnctd
    nn=length(resid);
    bin=resid_set{nn};
    [temploc tf]=isIn(bin, resid);
    temploc=[nn temploc];
    if not(tf)
        bin = [bin {resid}];
        if iscell(bin)
            gg=length(bin);
        else
            gg=1;
        end
        corecount=[corecount; [nn,gg]];
    end
end

```

```

        resid_set{nn}=bin;
        tempcopy=enumHelp(graph, coresize, resid);
        copy=[copy; tempcopy];

    elseif not(ismember(temploc,loc,'rows'))
        loc=[loc; temploc];

    end
    %end
end
end

%
% Yan Zhou, Zach Rubenstein
%
% findAnnihilateSet.m
%
% This function returns the annihilation set in the order that we
% annihilated them and its complement and a mutation of the original
% graph with all edges in the annihilation set removed.
%
function [f fc] = findAnnihilateComp(A, k, activenodes,...
    startNodeIndex,ff)
f = activenodes;
r = startNodeIndex;
fc=[];
while not(or(isempty(r),isempty(f)))
    fc=[fc ff(f(r))];
    f(r) = [];
    r = find(sum(A(f,f)) < k);
end
end

function [f tf] = findAnnihilateComp2(A, k, activenodes,...
    startNodeIndex,checkIndex)
f = activenodes;
r = startNodeIndex;
F=[];
tf=0;
F(activenodes(checkIndex))=0;
l=length(checkIndex);
while not(or(isempty(r),isempty(f)))

```



```

F(f(r))=1;
for j=1:l
    if F(activenodes(checkIndex(j)))==1
        f=[];
        tf=1;
        return
    end
end
f(r) = [];
r = find(sum(A(f,f)) < k);
end
end

```

```

function tf=checkcore(A,k,afc)
f = afc;
r = find(sum(A(f,f)) < k);
while not(or(isempty(r),isempty(f)))
    f(r) = [];
    r = find(sum(A(f,f)) < k);
end
tf=not(isempty(f));
end

```

```

%Vector of neighbor indices for node
function n = N(node, A)
n = find(A(node,:));
end
%Degree of node
function d = D(node, A)
d = sum(A(node,:));
end

```

```

function [loc tf] = isIn(array, target)
loc=[];
tf = 0;
if isempty(target)
    tf = 1;
    return
end
if isempty(array)
    tf=0;
    return
end
if not(iscell(array))
    if all(array == target)

```

```
        tf=1;
        loc=1;
        return
    end
else
    for cellind = 1:length(array)
        curvector = array{cellind};
        if all(curvector == target)
            tf = 1;
            loc = cellind;
            return
        end
    end
end
end
end
```